Data-driven Closure for Fluid Models of Hall Thrusters

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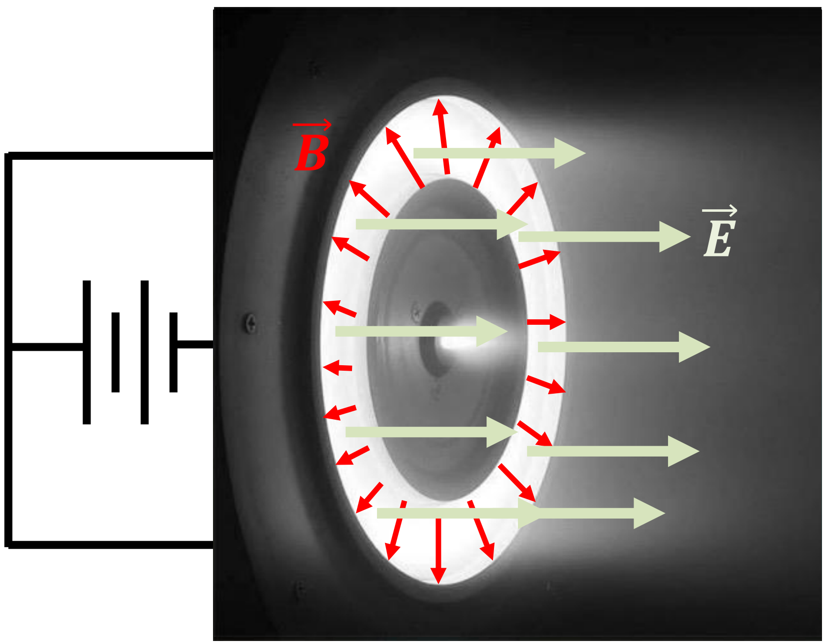
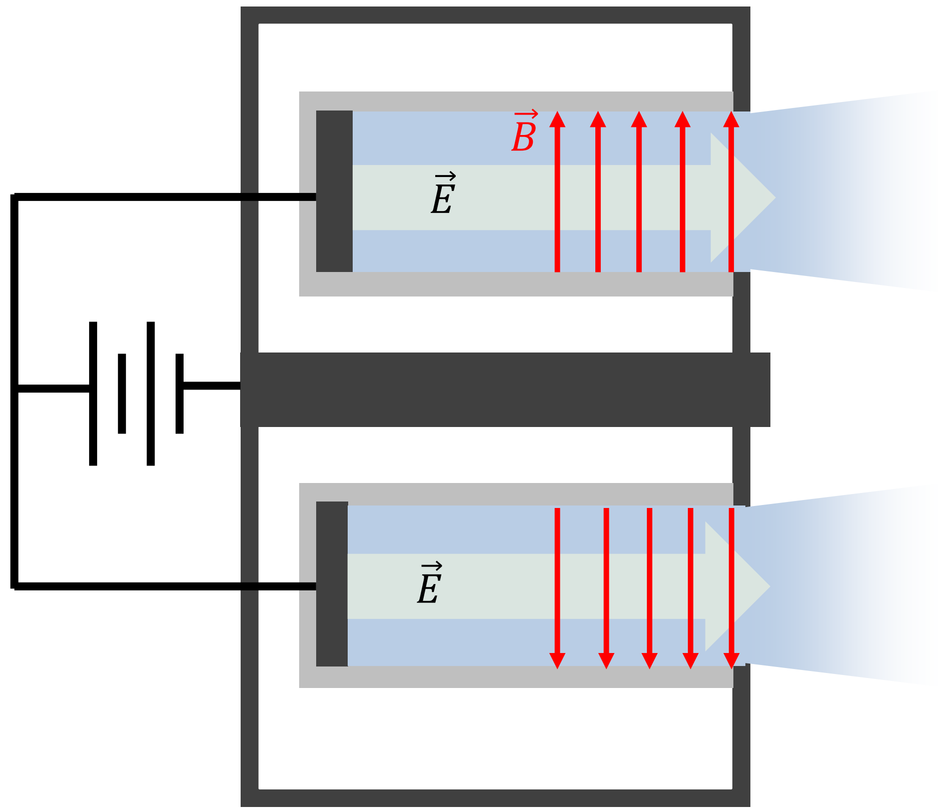
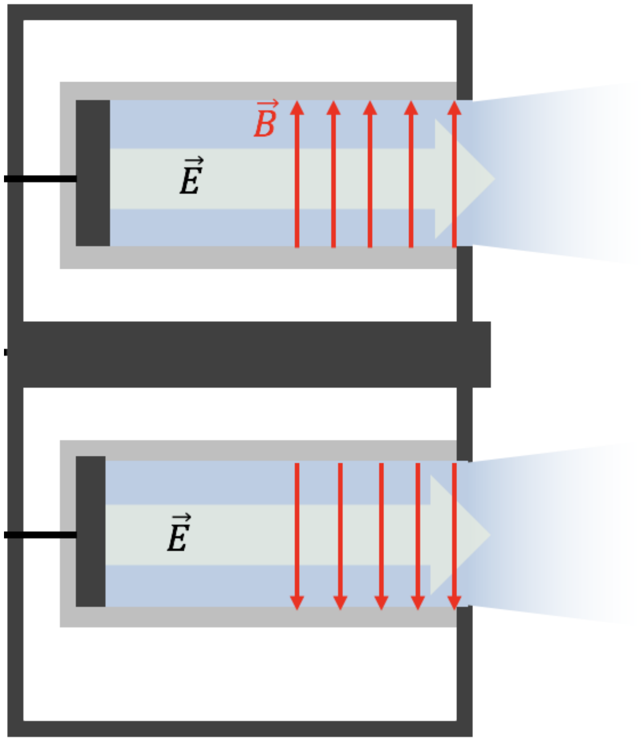
Princeton University ExB Workshop

The Hall effect thruster for space propulsion





Problem of anomalous electron transport



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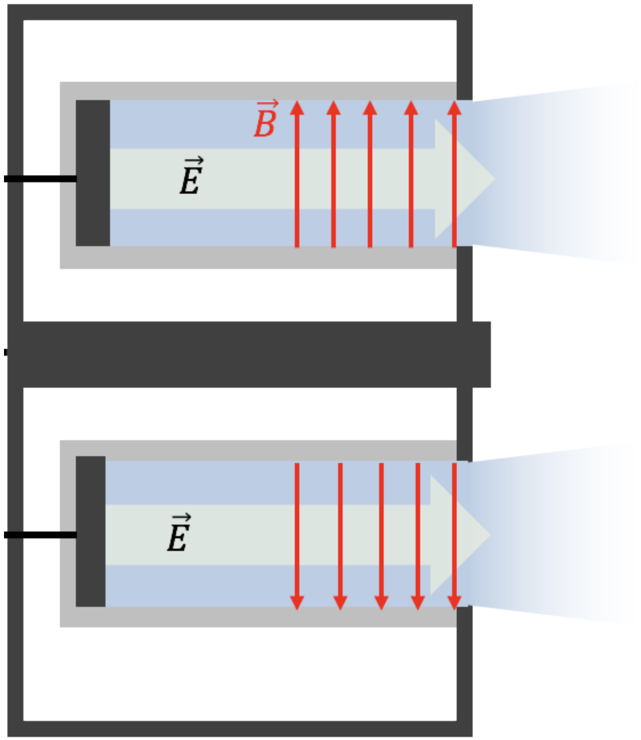


Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u_e} = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u_e} \times \vec{B}$$

Electron Energy

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e \quad + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

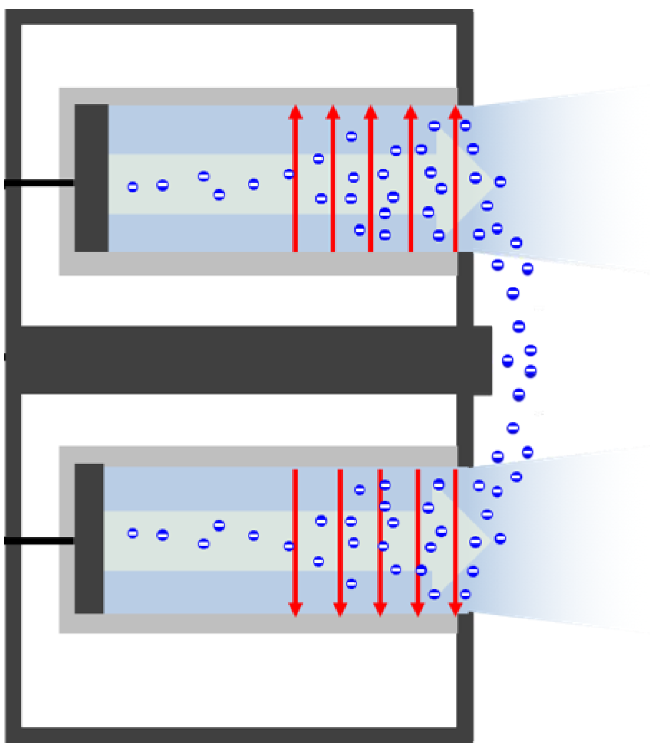
Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Closed set of classical equations that can be evaluated with standard techniques

Problem of anomalous electron transport

$I_e / I_d \sim 0.1\%$



Electron cross-field current from evaluating classical equations

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

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$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e \quad + \boldsymbol{Q_e} \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

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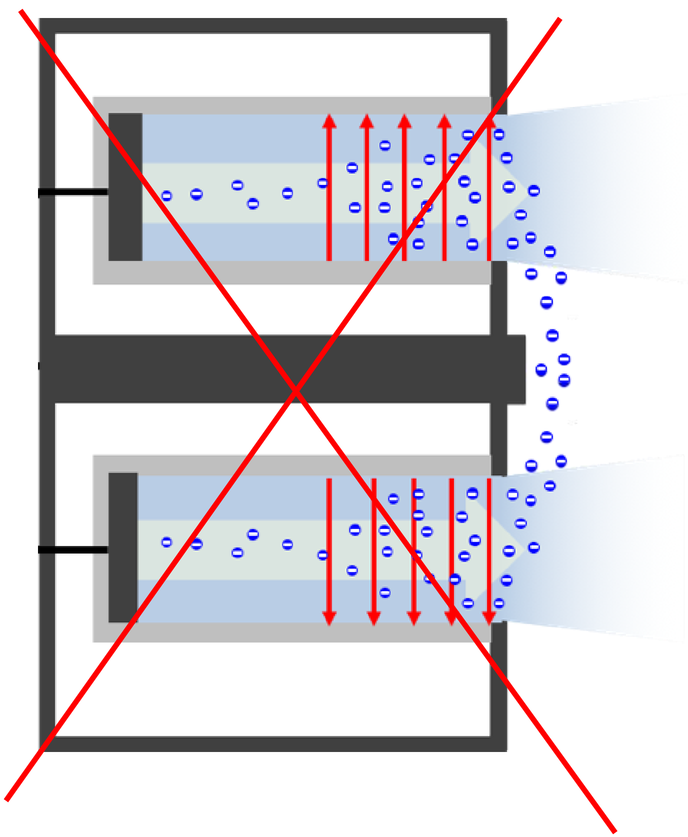

Actual cross-field current from evaluating equations 1000 x higher!

Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

Ohm's Law

$$\nu_e m_e n_e \boldsymbol{u}_e = -q n_e \vec{E} - \nabla P_e - q n_e \boldsymbol{u}_e \times \vec{B}$$

Electron Energy

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q n_e \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Problem of anomalous electron transport

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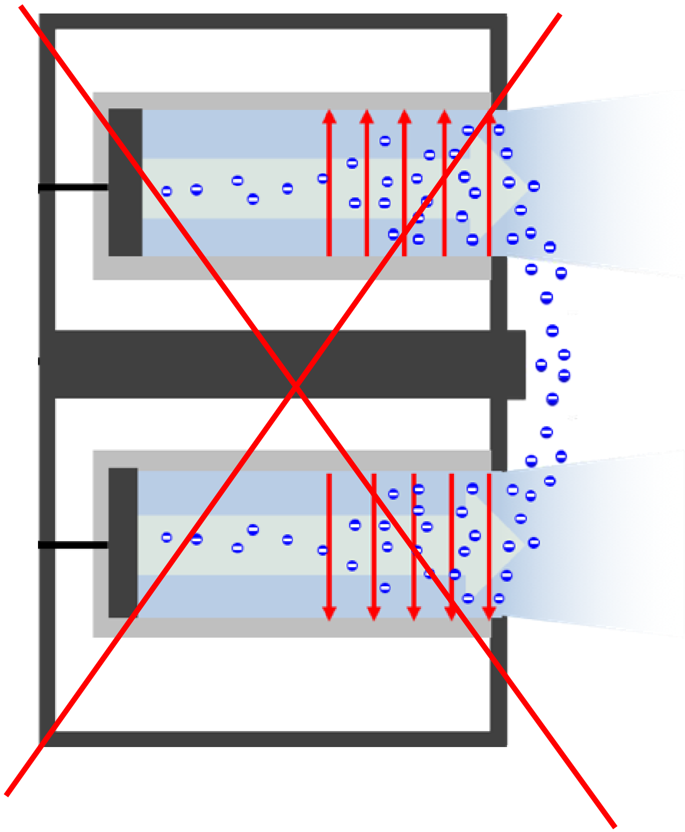

Actual cross-field current from evaluating equations <u>1000 x higher</u>!

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$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

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Electron Energy

Need to introduce ad hoc factor

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{\ } \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

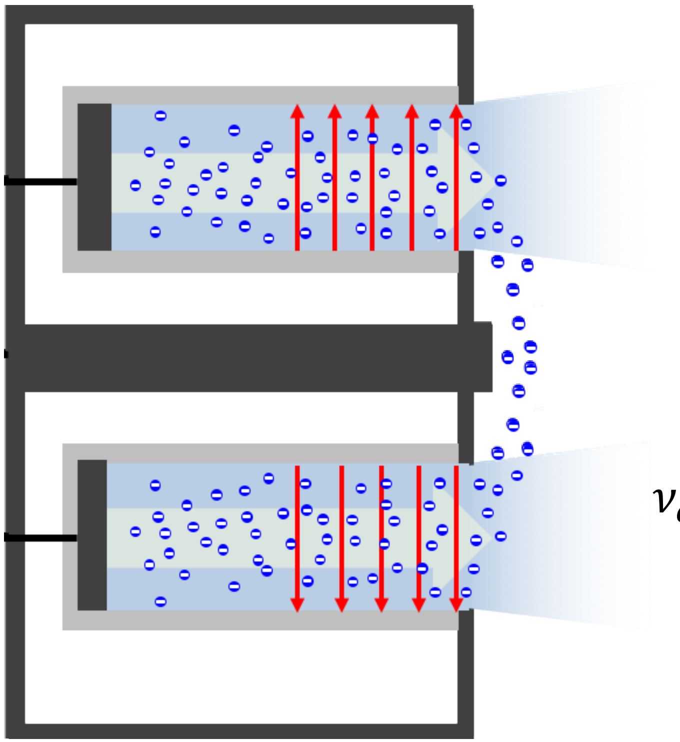
"Anomalous collision frequency"

Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Problem of anomalous electron transport

$I_e / I_d \sim 10\%$



Ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{u}_i) = 0$$

Ion momentum

$$\frac{\partial (m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

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Need to introduce ad hoc factor

"Anomalous collision frequency "

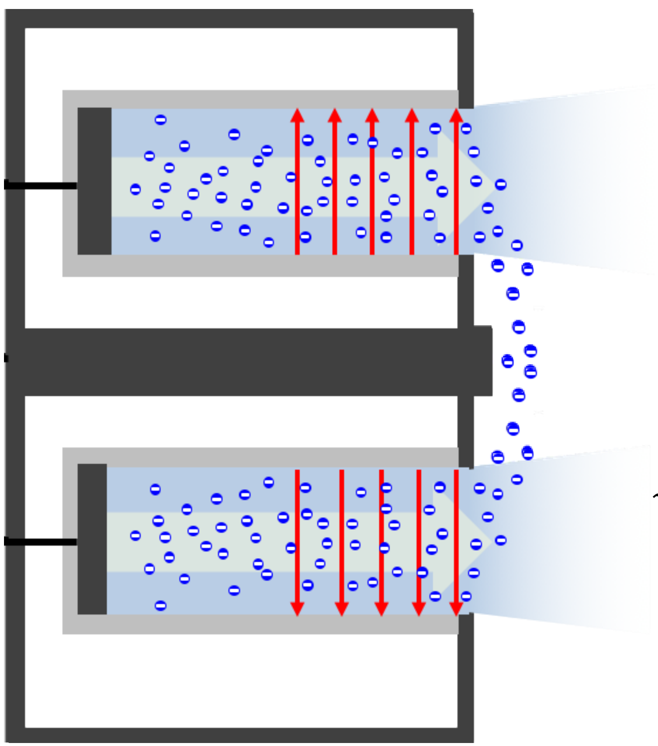
Current conservation

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

Anomalous friction term promotes additional cross-field current

Problem of anomalous electron transport

$I_e / I_d \sim 10\%$



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Electron Energy

Need to introduce ad hoc factor

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} = -q \frac{\rho_i}{m_i} \boldsymbol{E} \cdot \boldsymbol{u}_e - \nabla \cdot \left(\frac{5}{2} n_e T_e \boldsymbol{u}_e + \boldsymbol{Q}_e \right) + \frac{3}{2} T_e \nabla \cdot (n_e \boldsymbol{u}_e)$$

Current conservation

Problem: introducing ad-hoc term opens set of equations (too many unknowns)

$$0 = \nabla \cdot (q n_e [\boldsymbol{u}_e - \boldsymbol{u}_i])$$

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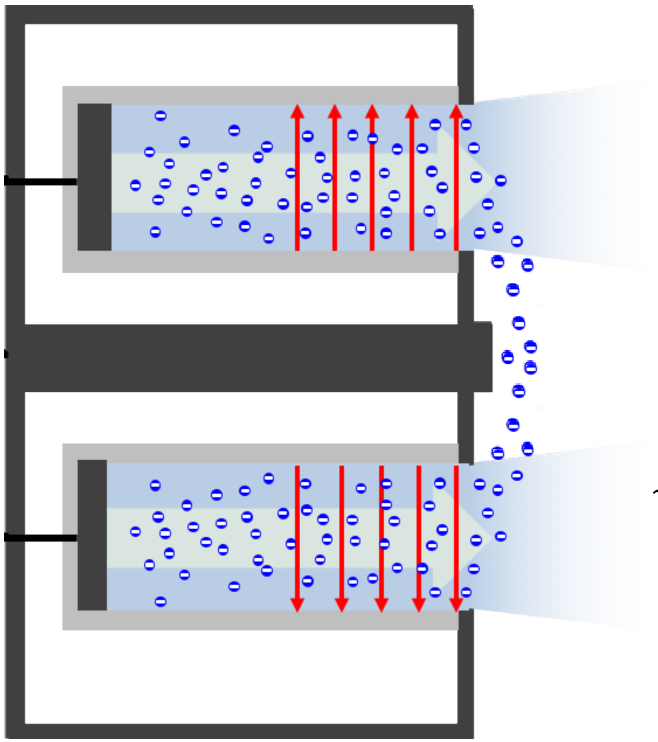

Anomalous friction term promotes additional cross-field current

Ion continuity

We need a functional form for $\nu_{AN}(T_e, n_e, ..)$ that depends on classical fluid parameters

Ion momentum

$$\frac{\partial(m_i n_i \boldsymbol{u}_i)}{\partial t} + \nabla \cdot (m_i n_i \boldsymbol{u}_i \boldsymbol{u}_i) = q\, n_i \boldsymbol{E} - \nu_i m_i (\boldsymbol{u}_i - \boldsymbol{u}_e)$$

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Current conservation

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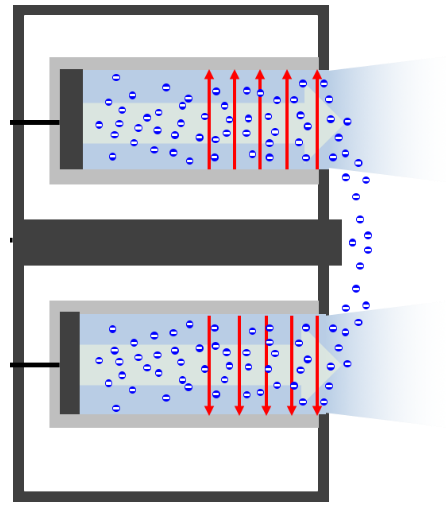
Closures for anomalous collision frequency: first-principles

$$\vec{F}_{AN} = -n_e m_e \, \nu_{AN} \boldsymbol{u}_e$$



*N. Gascon, M. Dudeck, and S. Barral, *PoP*, vol. 10, no. 10, 2003
† J. M. Fife and M. Martinez-Sanchez/ IEPC-95-24
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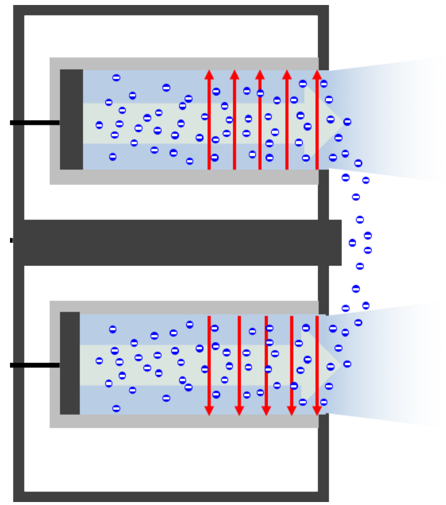

Wall Interactions*

$$\nu_{AN} = \beta\sqrt{T_e}$$

Instabilities‡

$$\nu_{AN} = \frac{1}{K}\,\omega_{ce}\left(\frac{v_{de}}{c_s}\right)^2$$

$$\nu_{AN} = \frac{|\nabla\cdot(\,\vec{u}_i n_e T_e)|}{m_e c_s n_e v_{de}}$$

Bohm Diffusion†

$$\nu_{AN} = \frac{1}{K}\,\omega_{ce}$$

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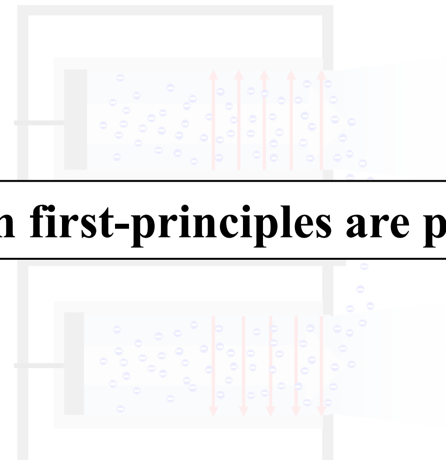

Wall B

$$\nu_{AN} = \beta\sqrt{T_e}$$

es‡

$$\nu_{AN} = \frac{1}{K}\omega_{ce}\left(\frac{v_{de}}{c_s}\right)^2$$

Bohm Diffusion†

$$\nu_{AN} = \frac{1}{K}\omega_{ce}$$

Closure models from first-principles are potentially predictive

Models have to date have had limitations, yielding qualitative agreement over only limited range of conditions

Possible that reality is too complicated or models or too reduced fidelity

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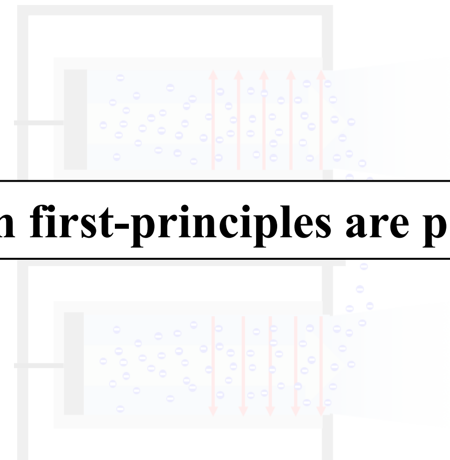

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Alternative: empirical form for collision frequency

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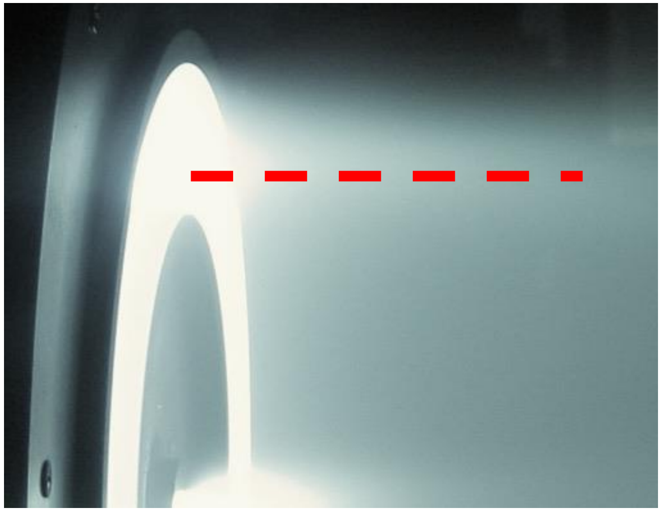
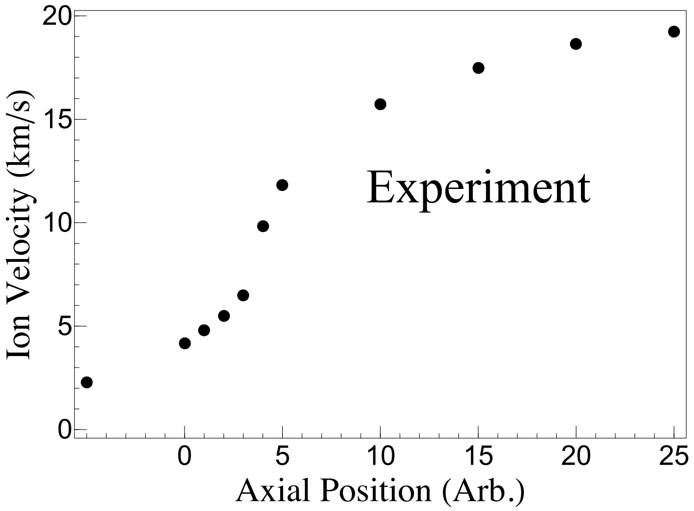
Closures for anomalous collision frequency: empirical estimate





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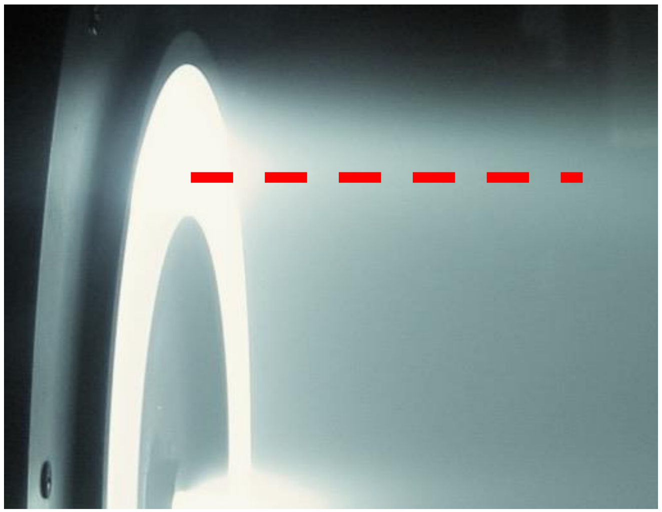

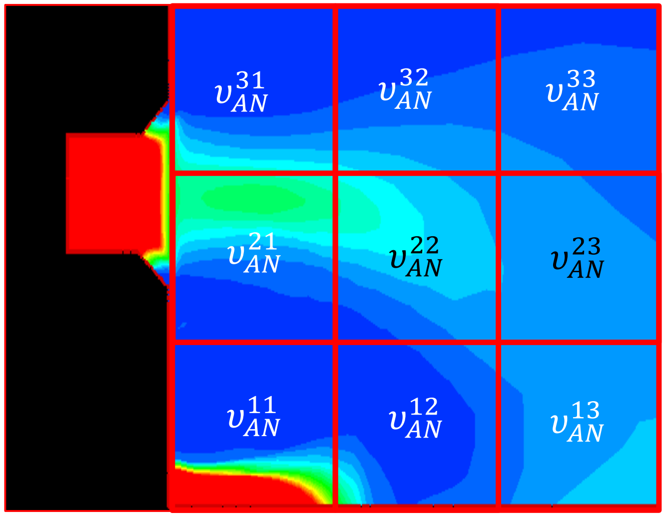

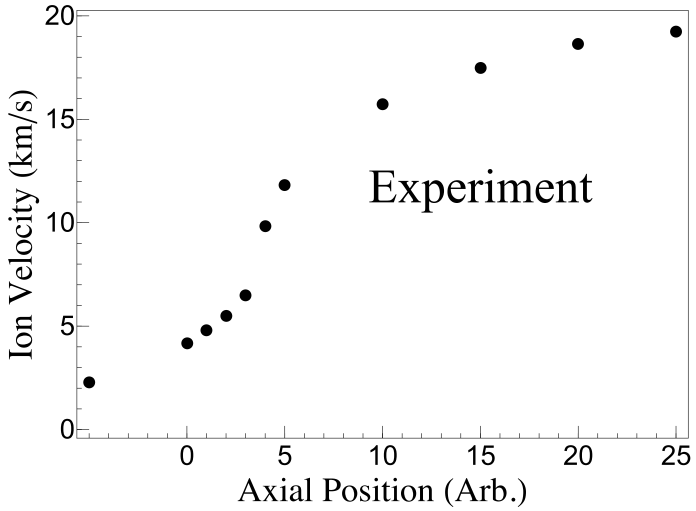

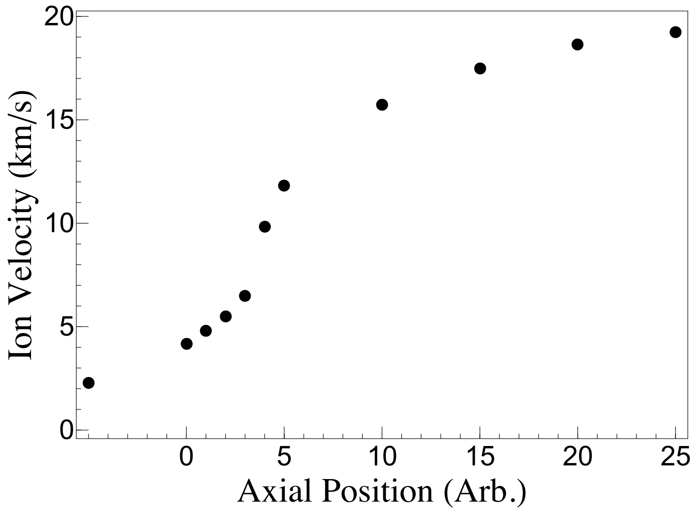

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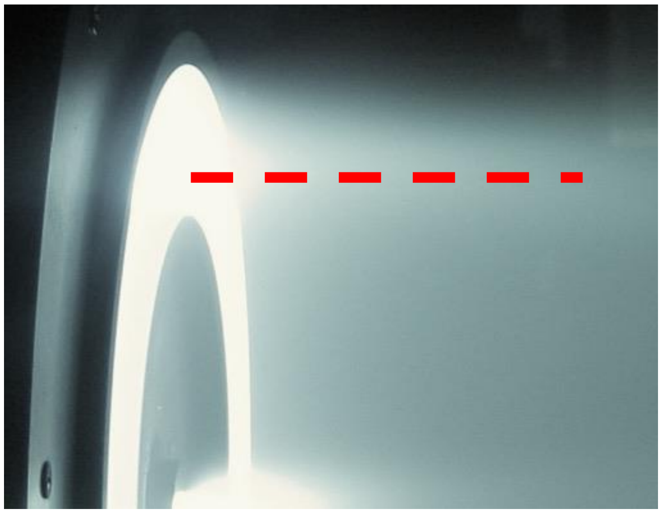
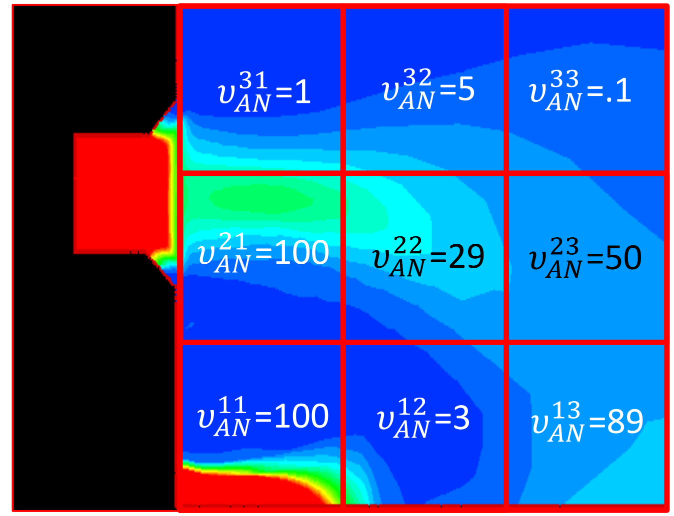
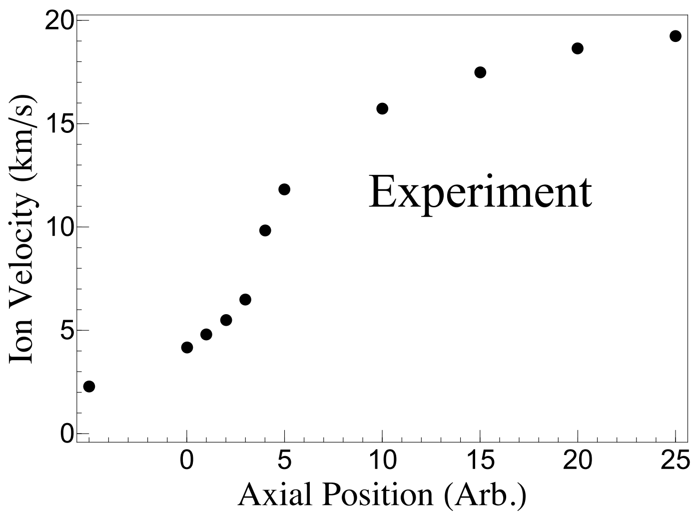
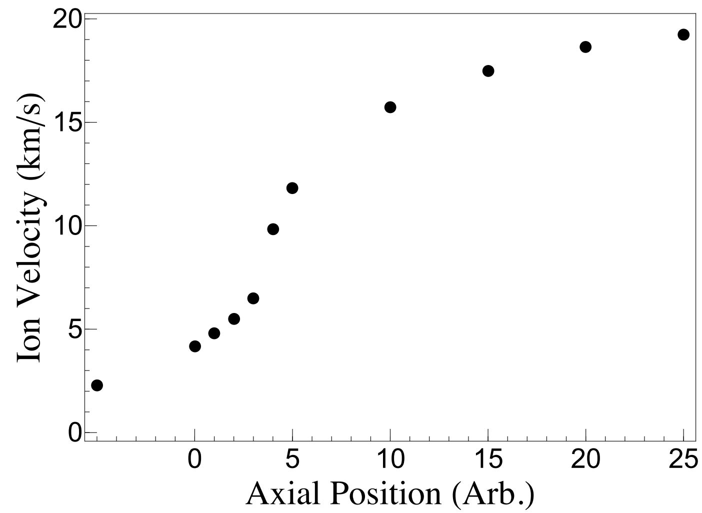


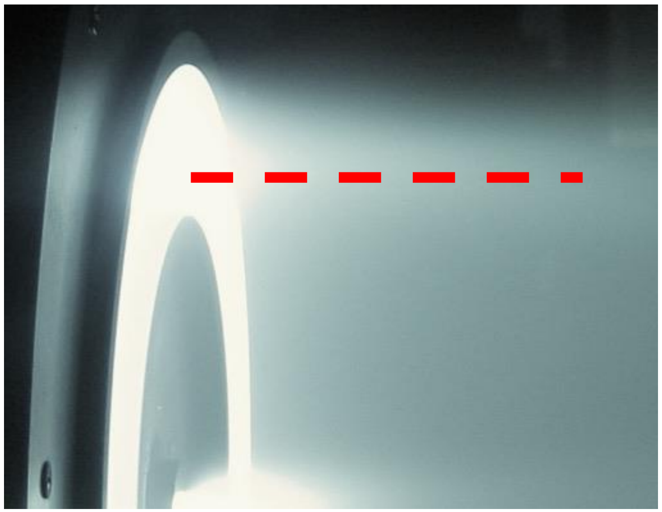


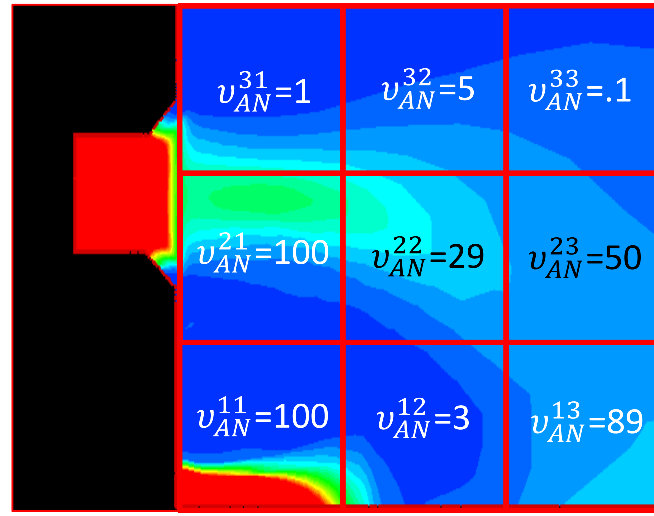


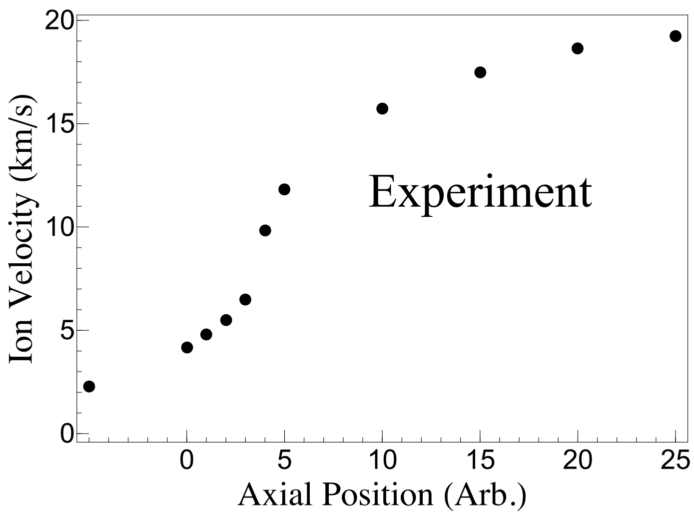
Iteration #1



Closures for anomalous collision frequency: empirical estimate







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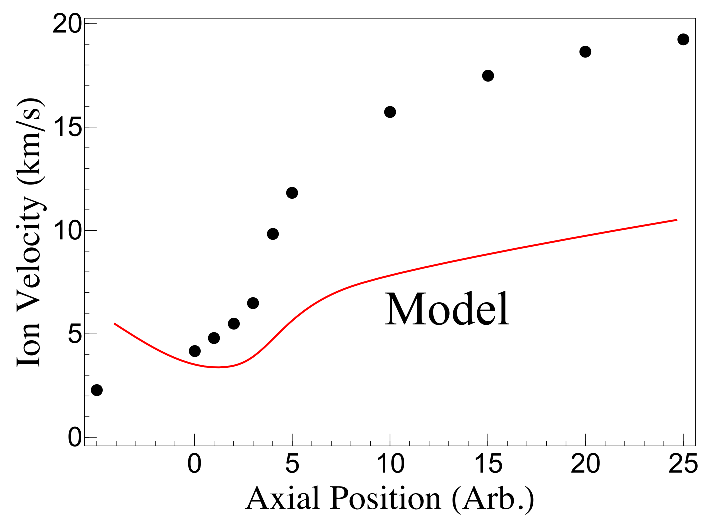
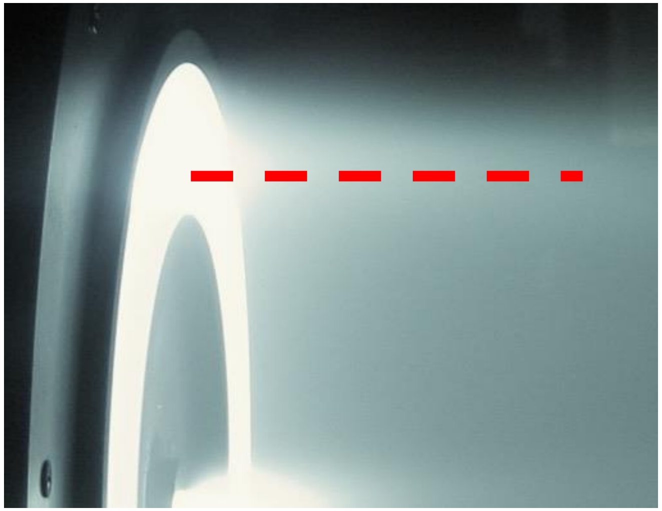
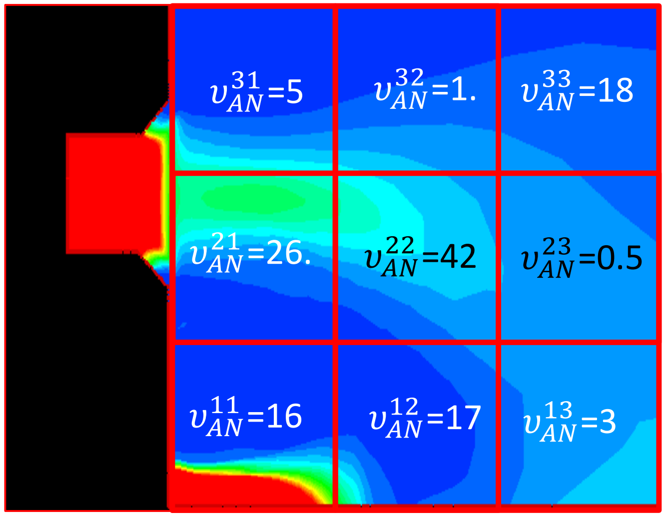
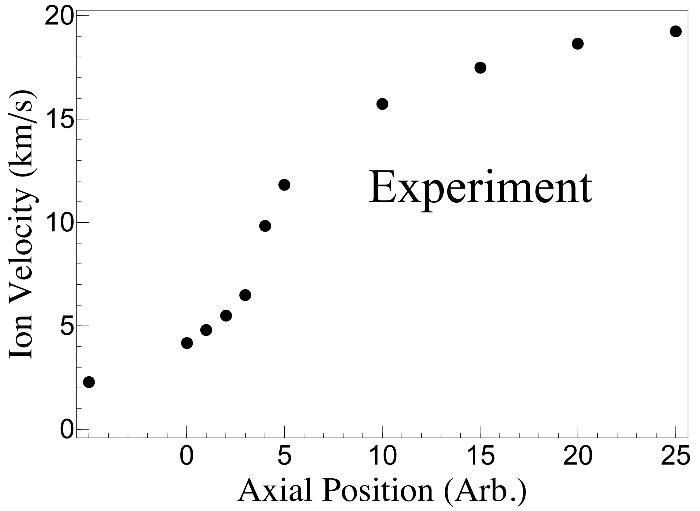


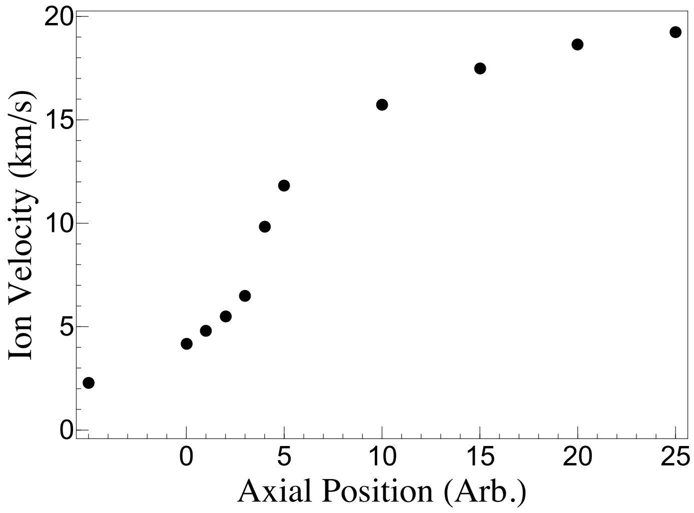
Closures for anomalous collision frequency: empirical estimate



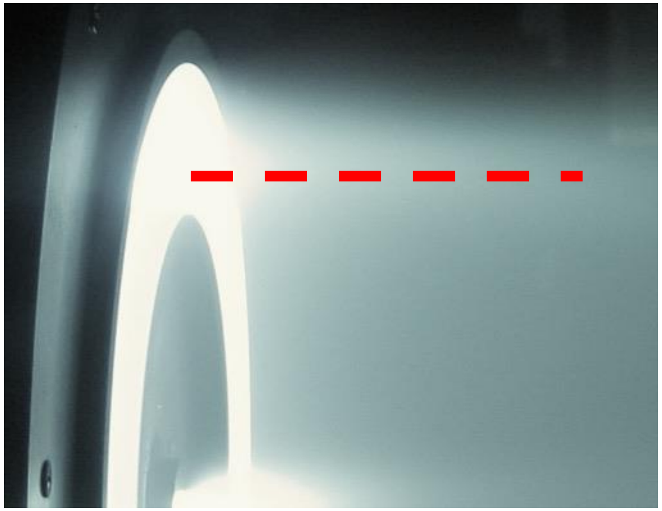


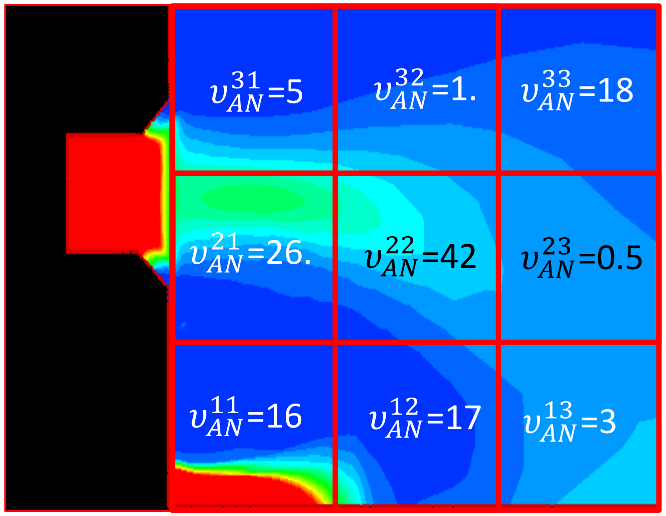


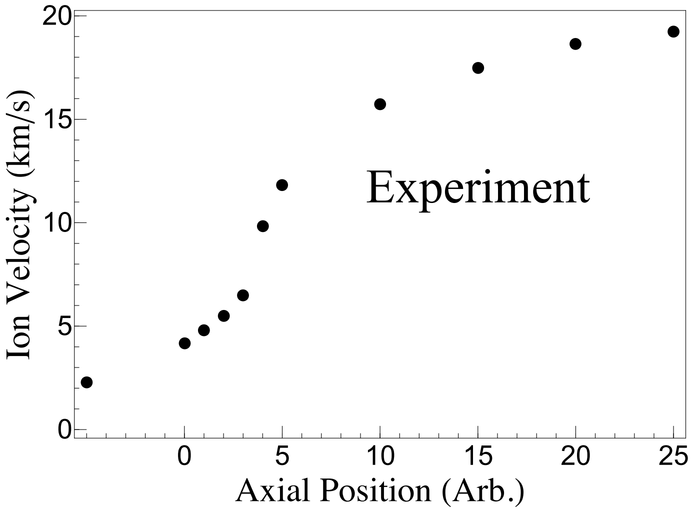
Iteration #2



Closures for anomalous collision frequency: empirical estimate







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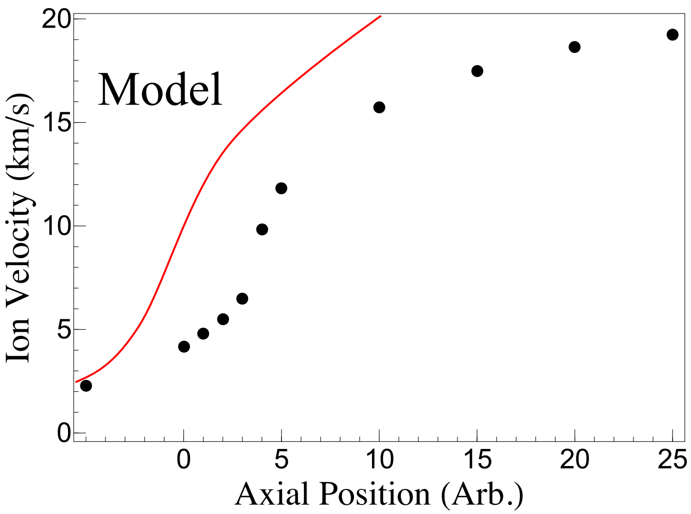

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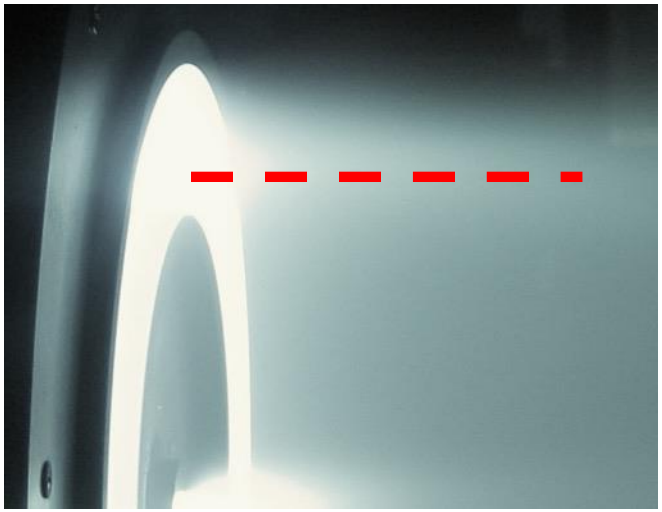
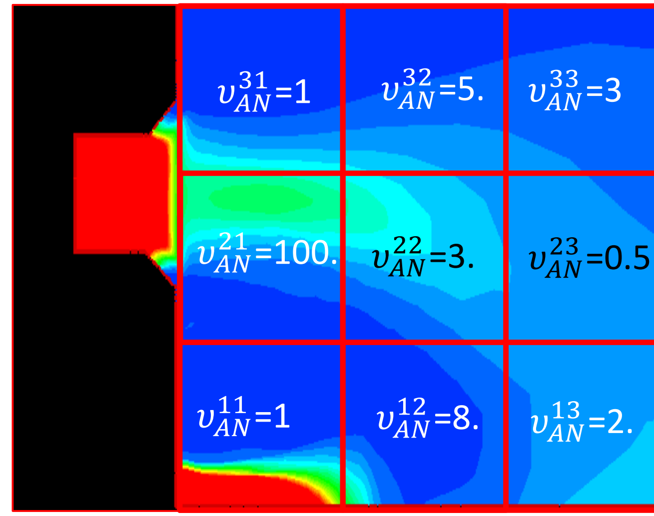
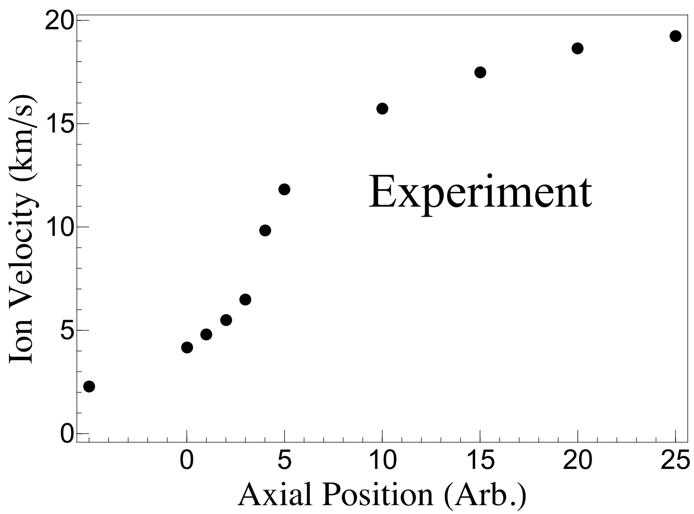
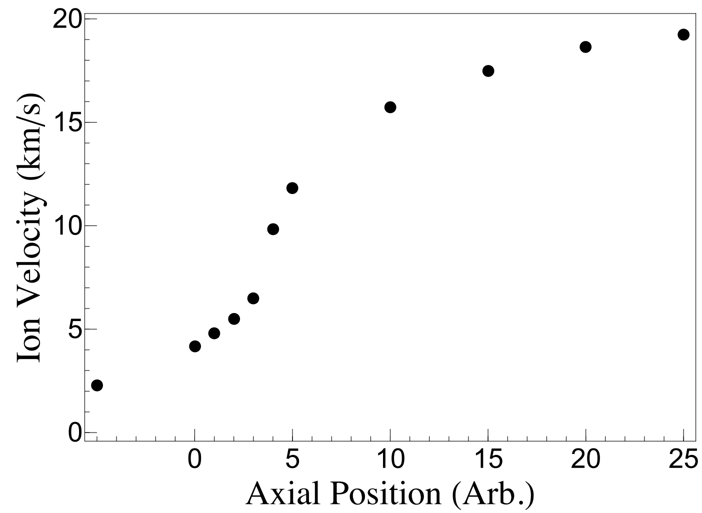


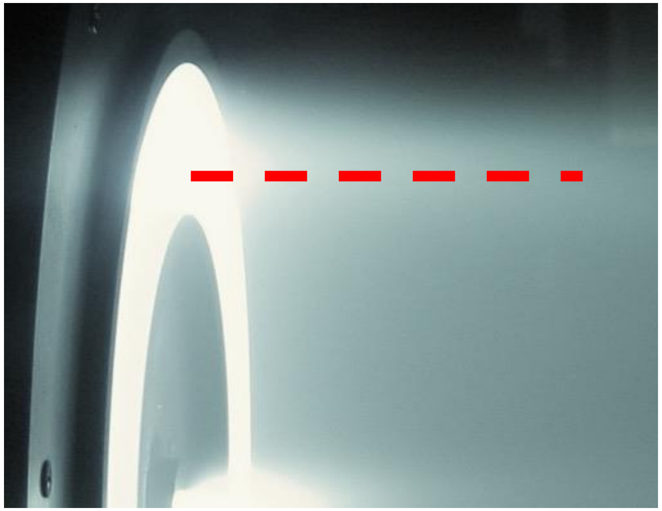


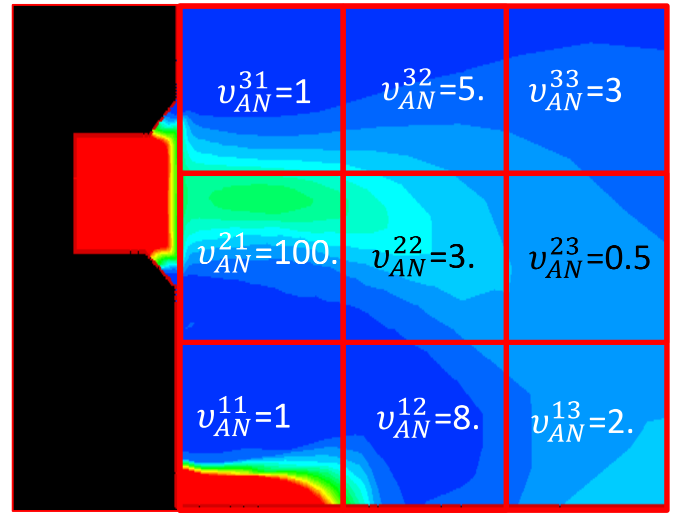


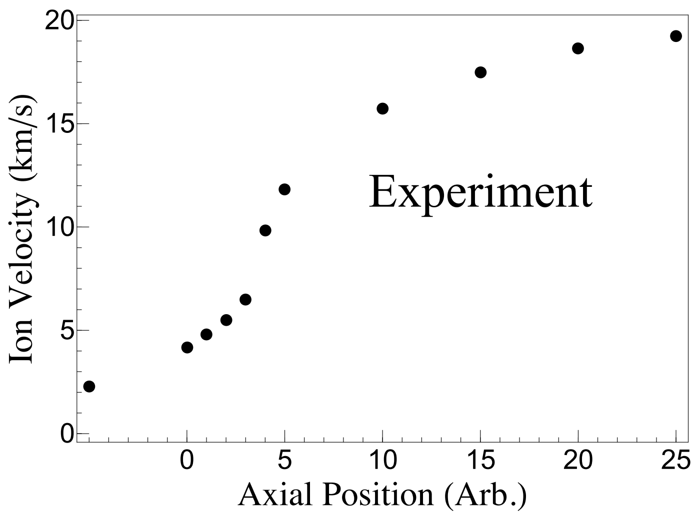
Iteration #3



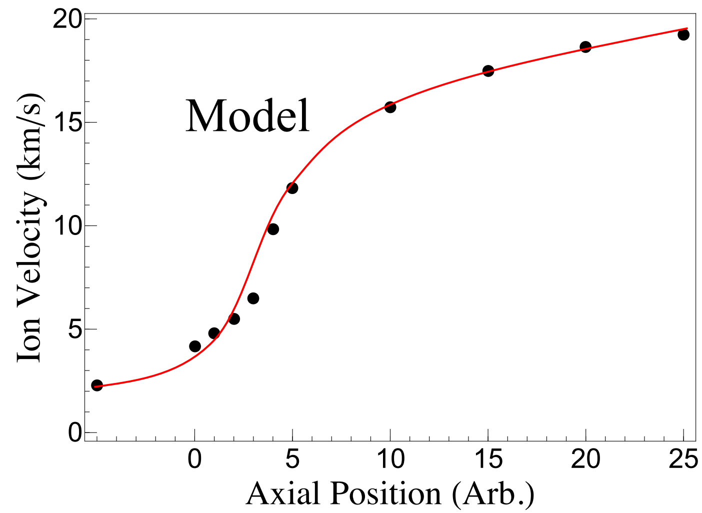
Closures for anomalous collision frequency: empirical estimate





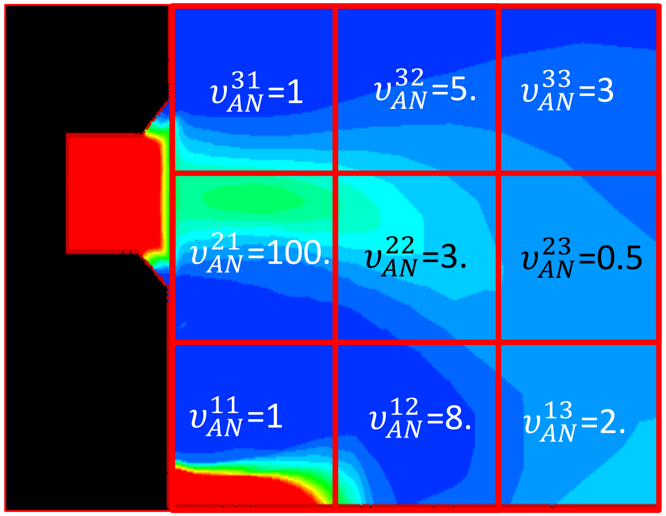


Iteration #3



Closures for anomalous collision frequency: empirical estimate

- Yields excellent agreement with experimental results for a given operating condition
- Collision frequency is specified empirically. Only applicable for data set used for validation



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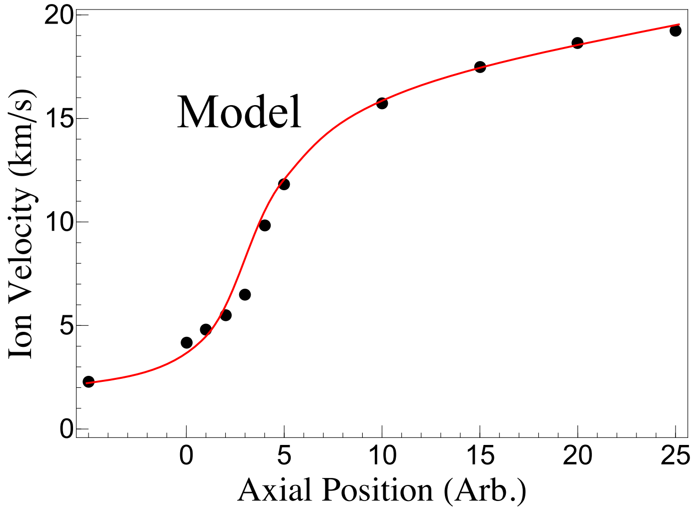

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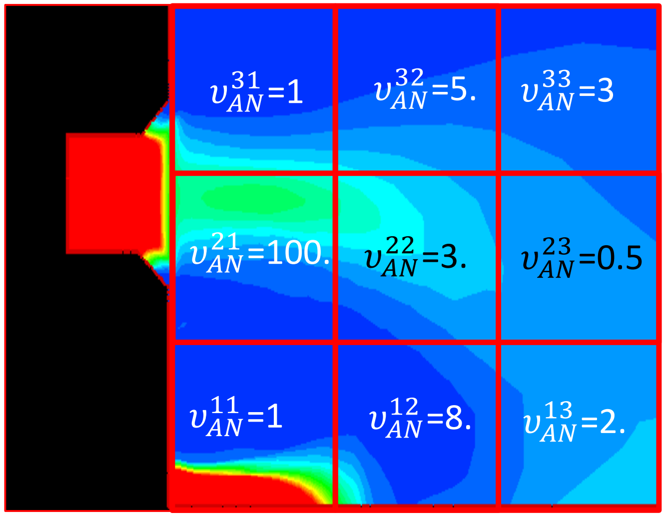

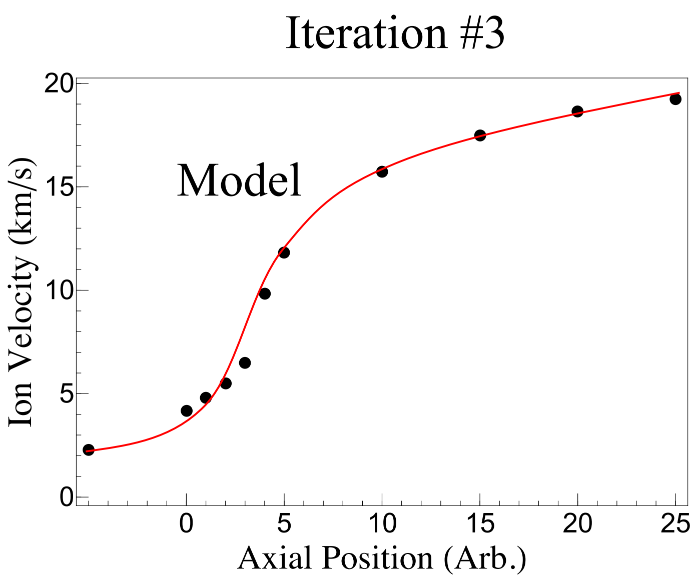

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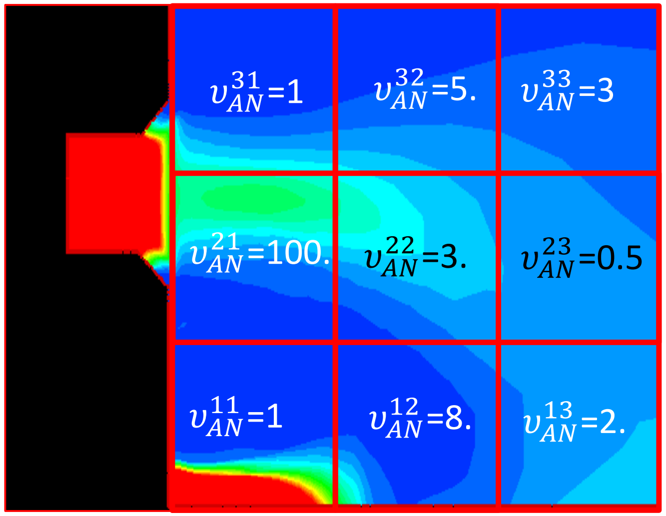
Hypothesis: we can use empirical data to generate a functional form, $\nu_{AN}(T_e, n_e, ..)$



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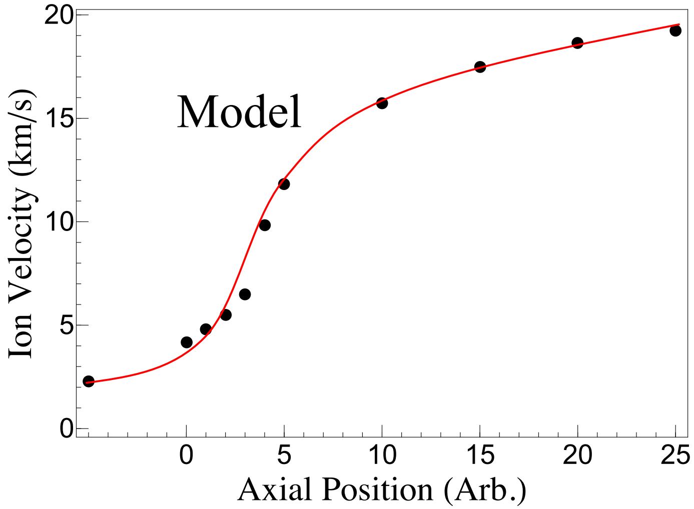
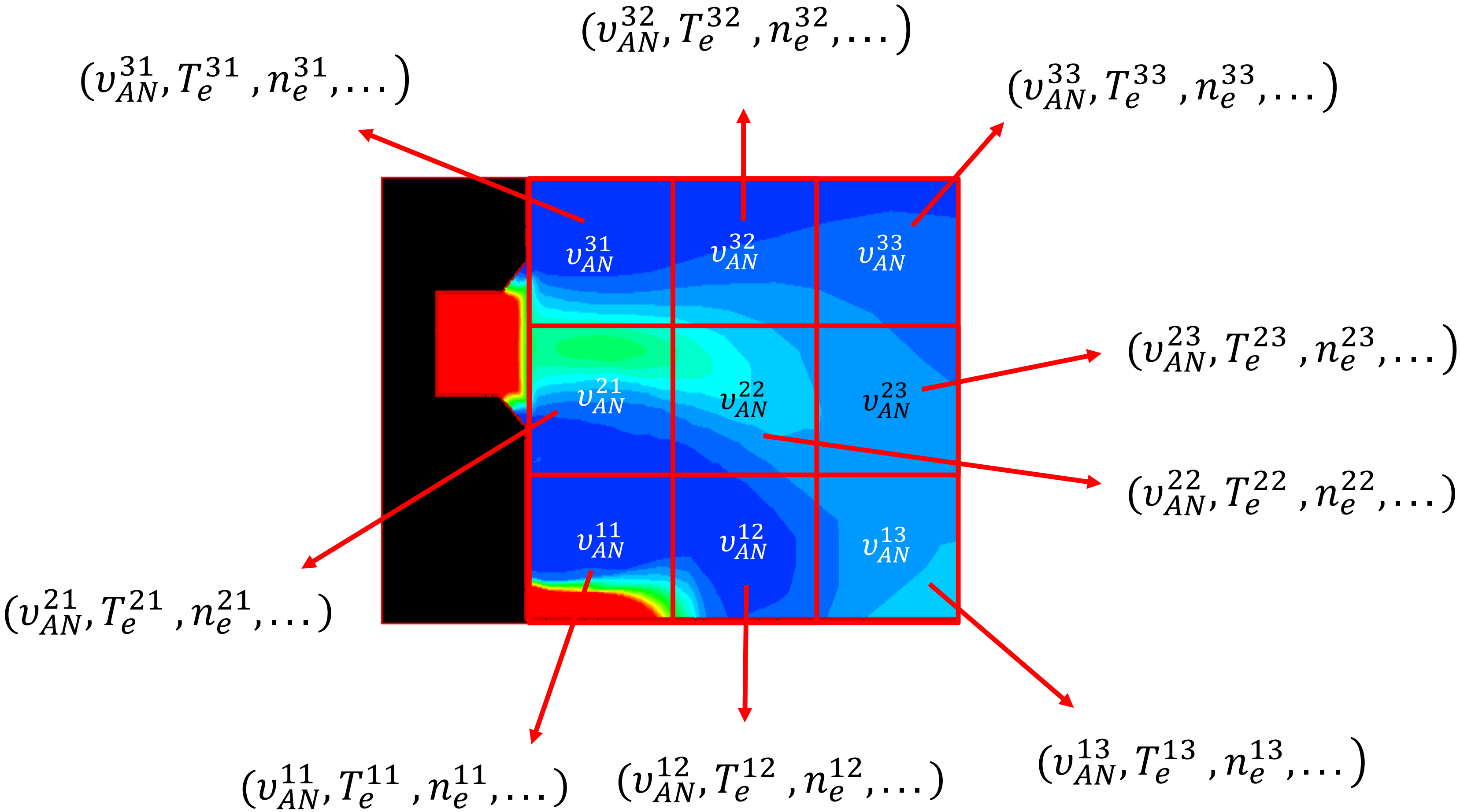
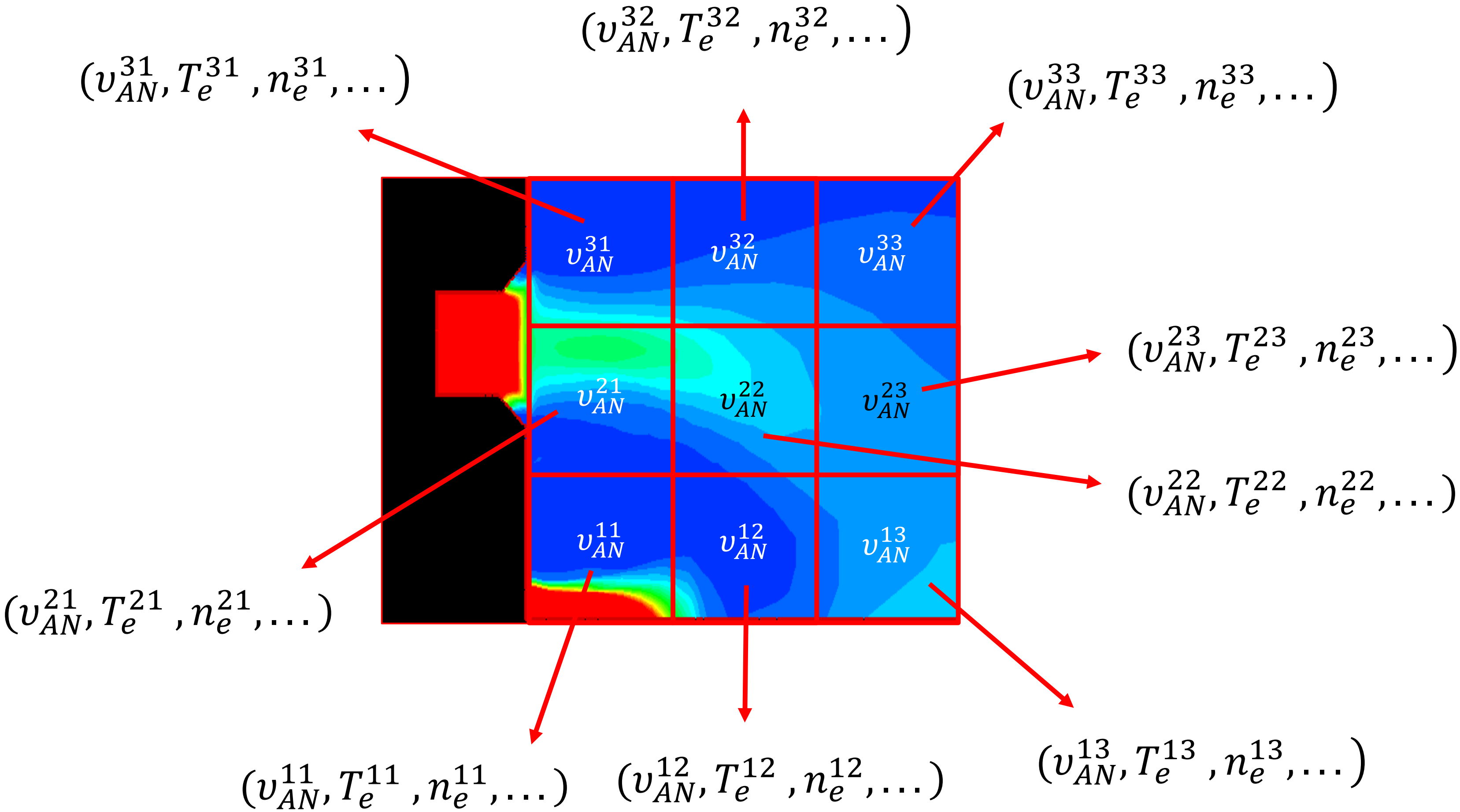


Model from regression



Each point from empirical model yields data point

Model from regression



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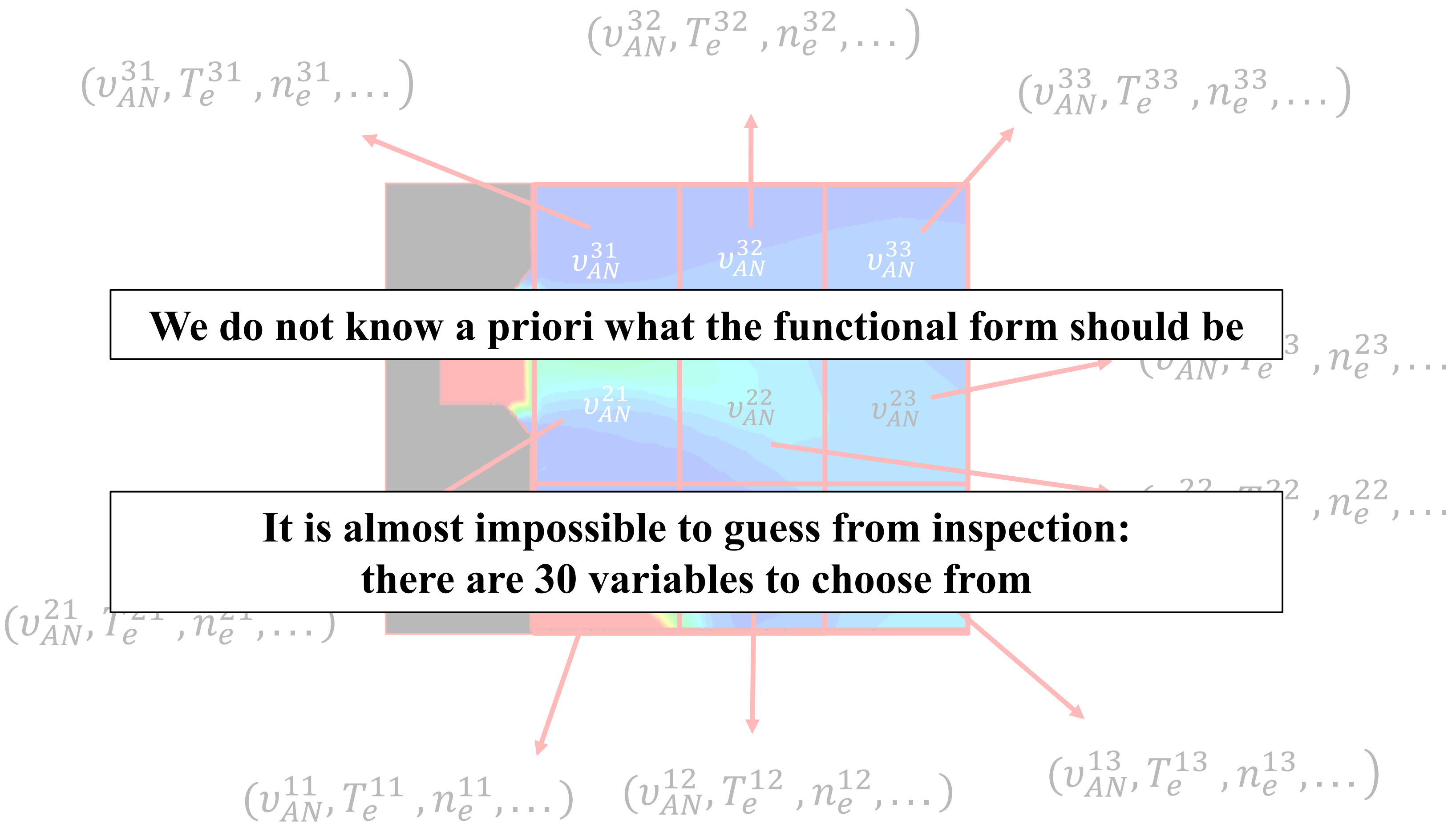

We do not know a priori what the functional form should be

It is almost impossible to guess from inspection: there are 30 variables to choose from

$$\nu_{AN} = c_1 T_e + c_2 (n_e)^2 + c_3 \mathrm{u_i}$$

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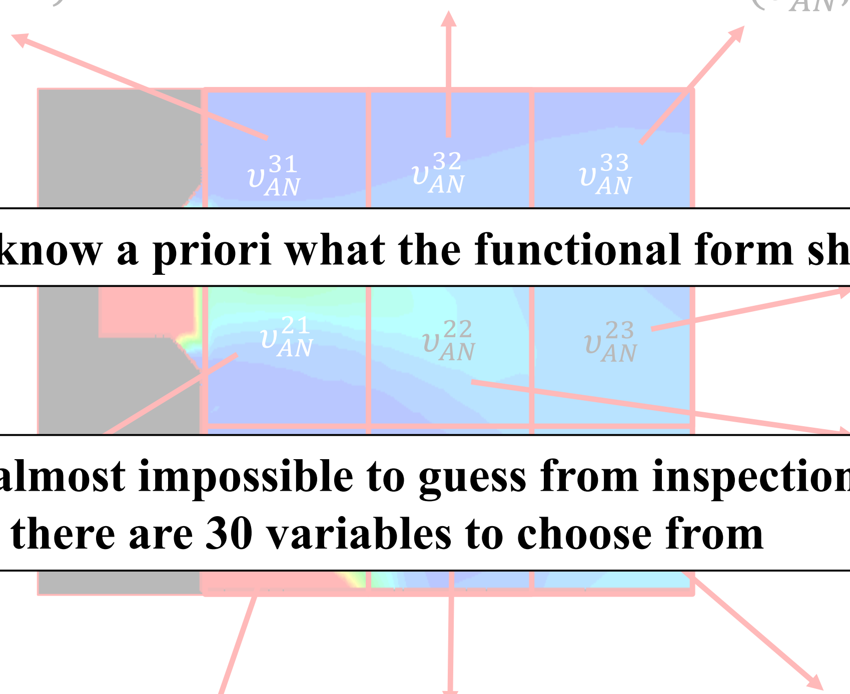


We do not know a priori what the functional form should be

It is almost impossible to guess from inspection: there are 30 variables to choose from

Solution: use machine learning to regress data

$$\nu_{AN} = c_1 T_e + c_2 (n_e)^2 + c_3 \mathrm{u_i}$$

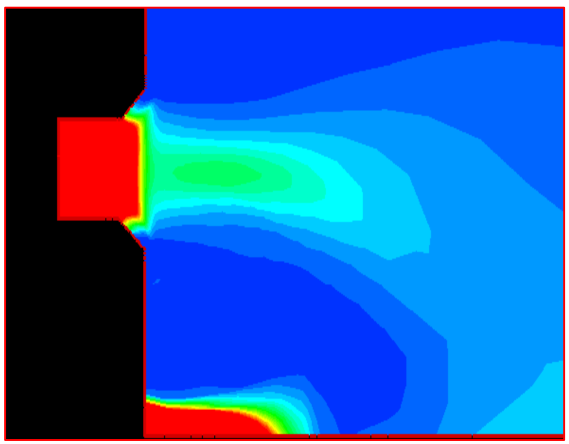
Maybe there is a function, $\nu_{AN}(T_e, n_e, \ldots)$, that fits the data

Regression with machine learning

*I. G. Mikellides and I. Katz, *Phys. Rev. E* vol. 86, no. 4, pp. 1–17, 2012.

Regression with machine learning

Generate datasets from empirically validated codes



7 operating conditions from 4 different thrusters from Hall2De*: 700 data points

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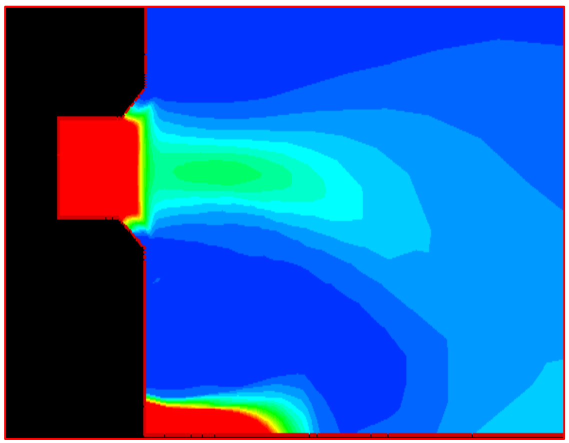

7 operating conditions from 4 different thrusters from Hall2De*: 700 data points

Prepare datasets for regression

Frequencies normalized by electron cyclotron frequency, ω_{ce}	
Ion plasma frequency	ω_{pi}
Classical electron collision frequency	f_e
Classical ion collision frequency	f_i
Velocities normalized by ion sound speed, c_s	
Ion axial velocity	u_i
Electron Hall velocity	v_{de}
Length scales normalized by electron Larmor radius, r_{ce}	
Debye length	λ_{de}
Pressure gradient length-scale	$L_P = P_e/\nabla P_e$
Ion drift velocity length-scale	$L_{ui} = u_i/\nabla u_i$

8 normalized lengthscales, velocities, and frequencies

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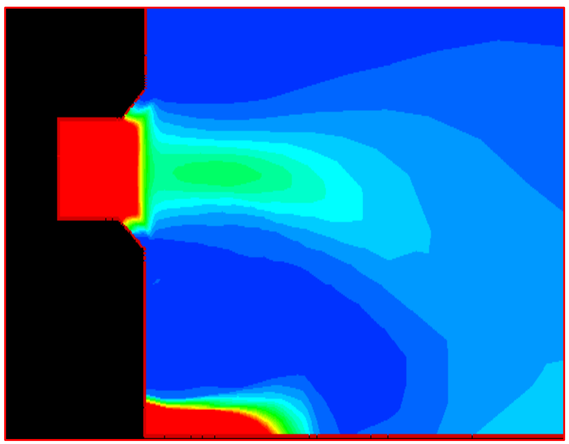

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8 normalized lengthscales, velocities, and frequencies

Apply ML regression algorithm

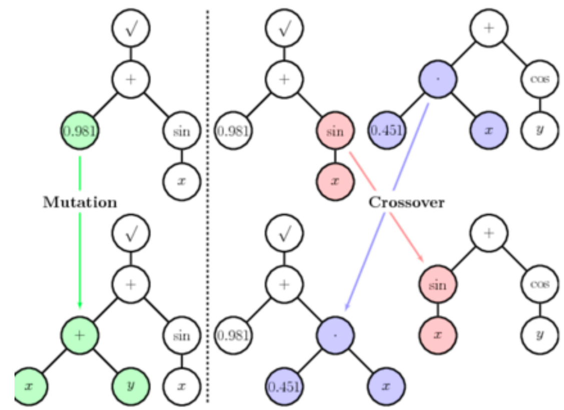
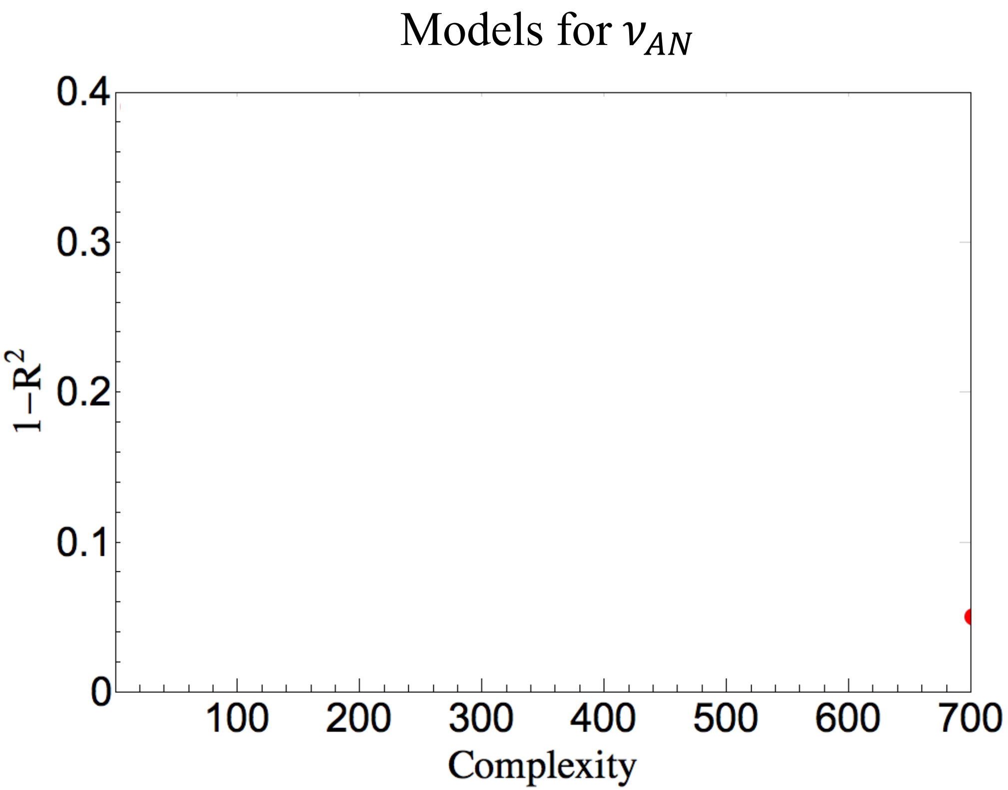


Image credit: M. Quade, Phys Rev. E. no 1. 2016

DataModeler symbolic regression from *Evolved Analytics*

*I. G. Mikellides and I. Katz, *Phys. Rev. E* vol. 86, no. 4, pp. 1–17, 2012.

Symbolic regression Pareto front



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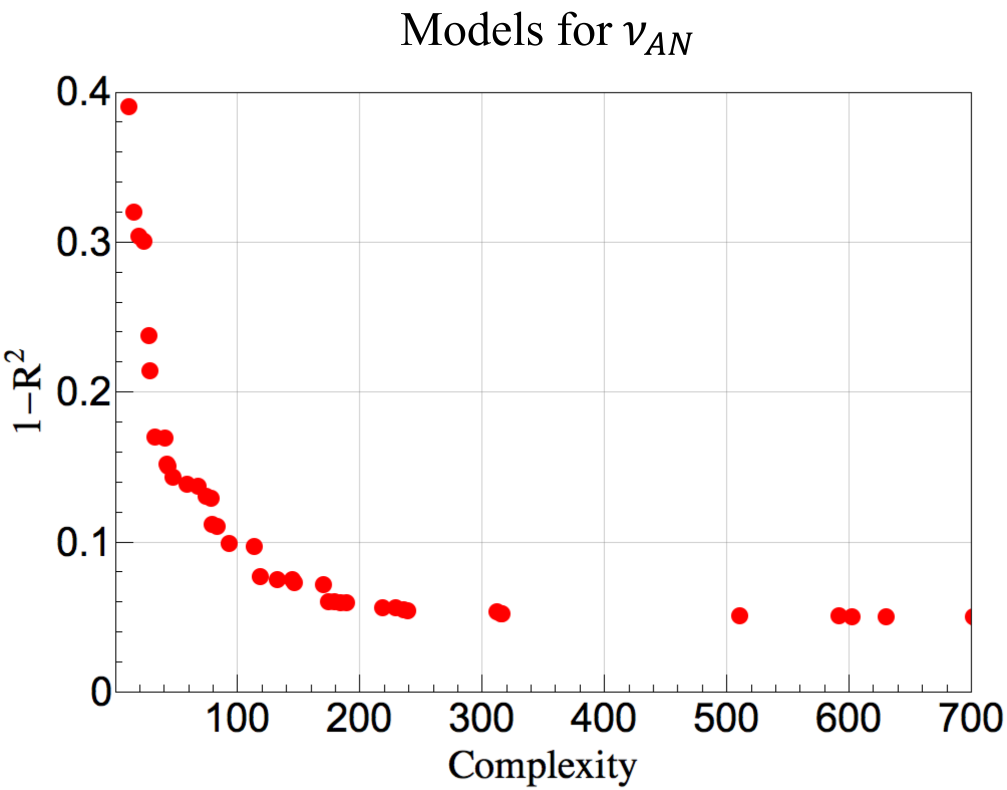
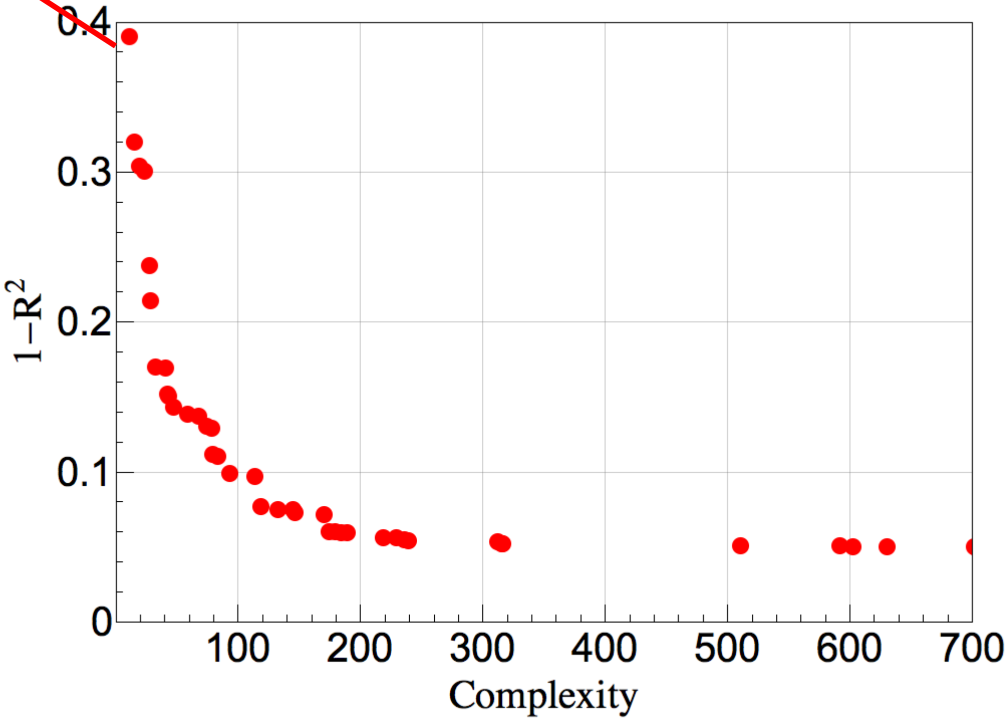


Symbolic regression Pareto front

$0.10018u_i - 0.049842$

Simple but poor fit to data

Models for ν_{AN}



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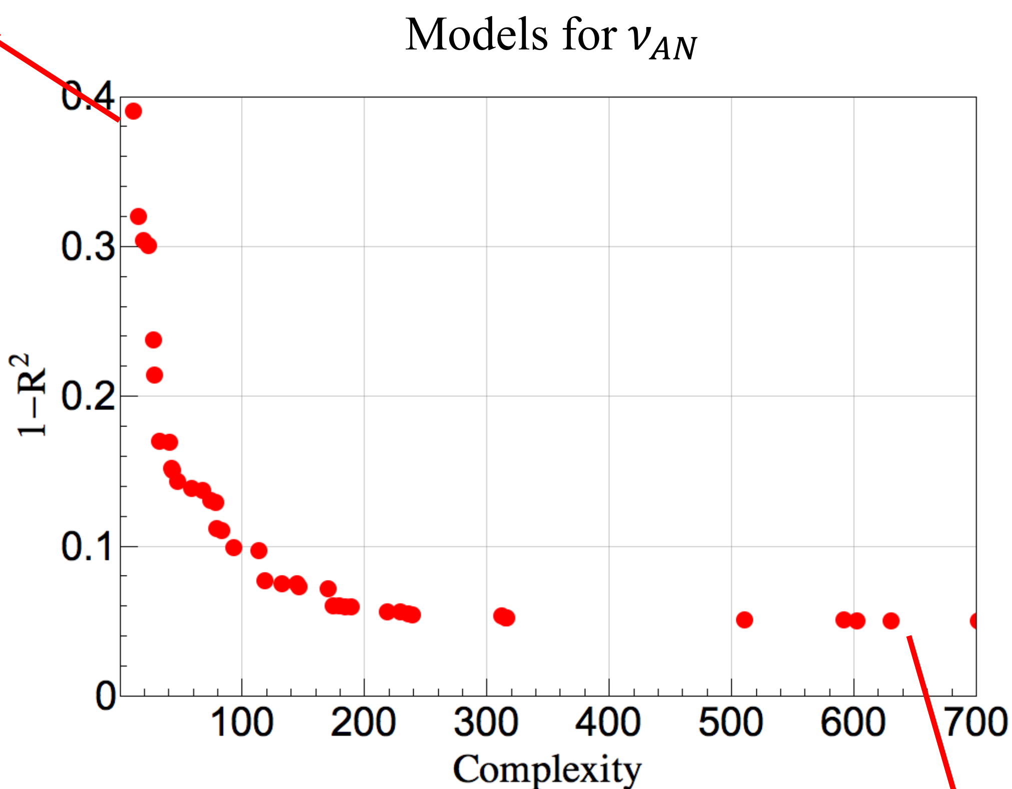

Complex and overfits data

$$\frac{5.87189u_i}{\frac{(v_{\mathrm{de}}-10)^4}{u_i^4} + \left(-u_i - \frac{\lambda_{\mathrm{de}}}{\sqrt{f_e}} + 10\right)^2 - (u_i-8)^2 + 4u_i - v_{\mathrm{de}} + \frac{\left(\frac{v_{\mathrm{de}}^2}{16} - u_i + \lambda_{\mathrm{de}} + \frac{4}{u_i\left(-u_i - \frac{\lambda_{\mathrm{de}}}{\sqrt{f_e}} + 10\right)} + 2.79118\right)^2}{u_i^2} + 23.6732}$$

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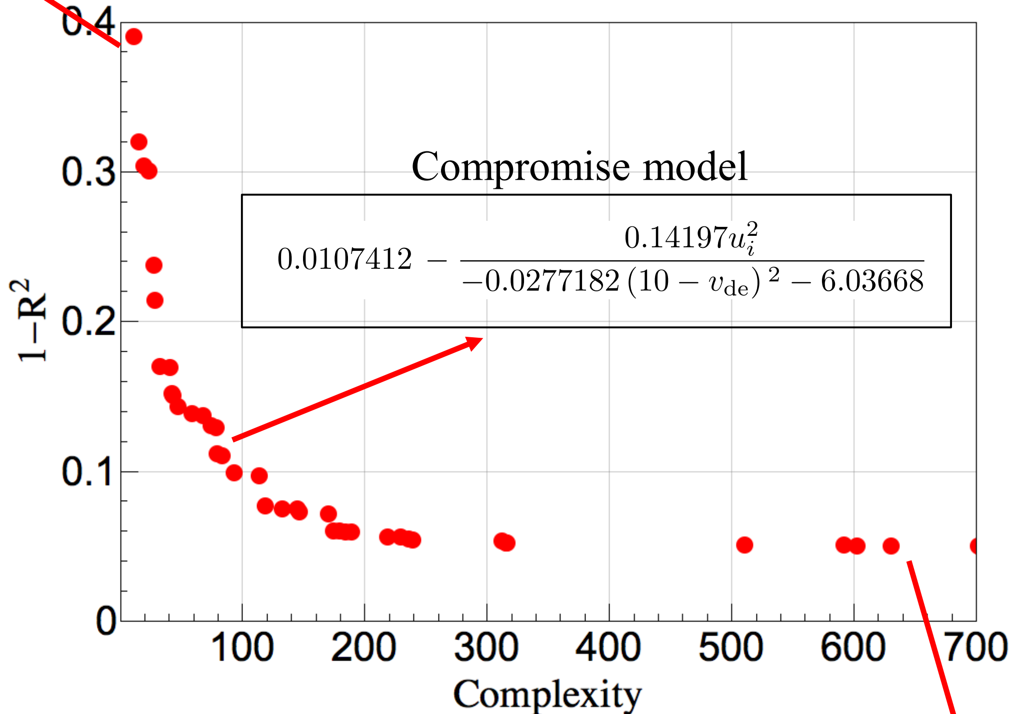

Compromise model

$$0.0107412 - \frac{0.14197u_i^2}{-0.0277182\,(10 - v_{\mathrm{de}})^2 - 6.03668}$$

Complex and overfits data

$$\frac{5.87189u_i}{\frac{(v_{\mathrm{de}}-10)^4}{u_i^4} + \left(-u_i - \frac{\lambda_{\mathrm{de}}}{\sqrt{f_e}} + 10\right)^2 - (u_i - 8)^2 + 4u_i - v_{\mathrm{de}} + \frac{\left(\frac{v_{\mathrm{de}}^2}{16} - u_i + \lambda_{\mathrm{de}} + \frac{4}{u_i\left(-u_i - \frac{\lambda_{\mathrm{de}}}{\sqrt{f_e}} + 10\right)} + 2.79118\right)^2}{u_i^2} + 23.6732}$$

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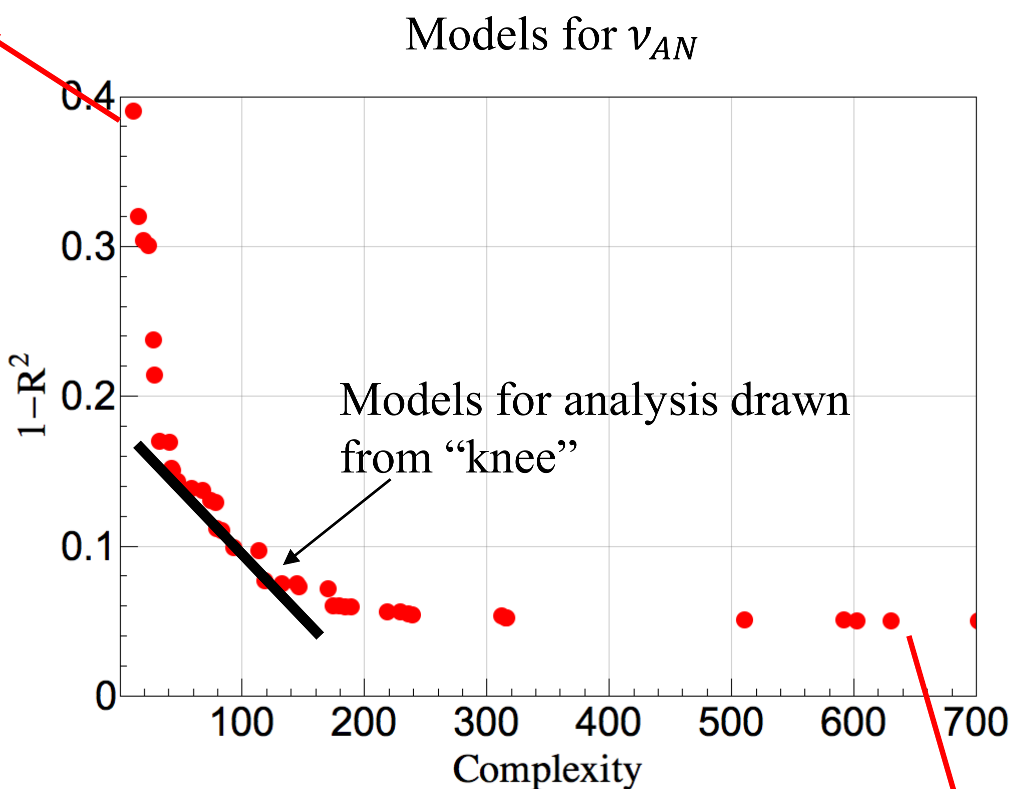

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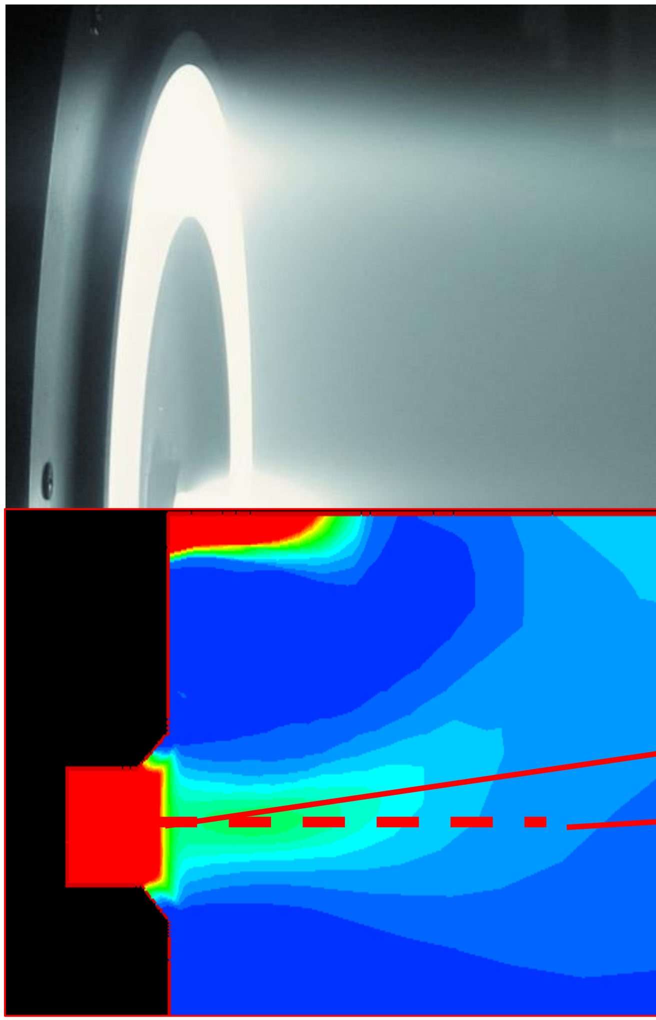
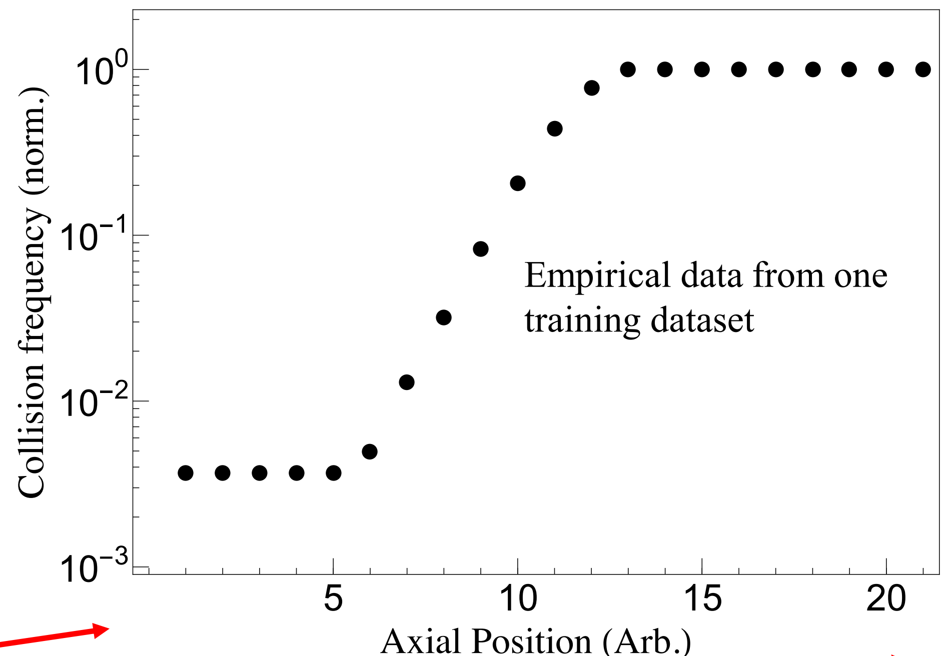
Comparison of models to training data

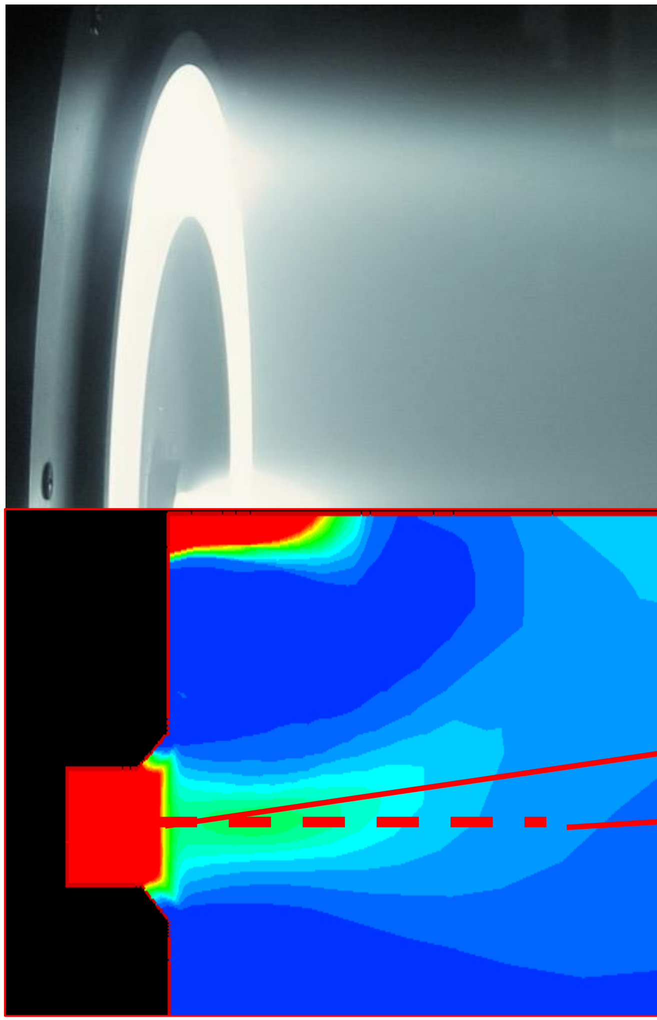


Normalized frequency on channel centerline



B. Jorns., *PSST,* **27** (10), 2018

Comparison of models to training data



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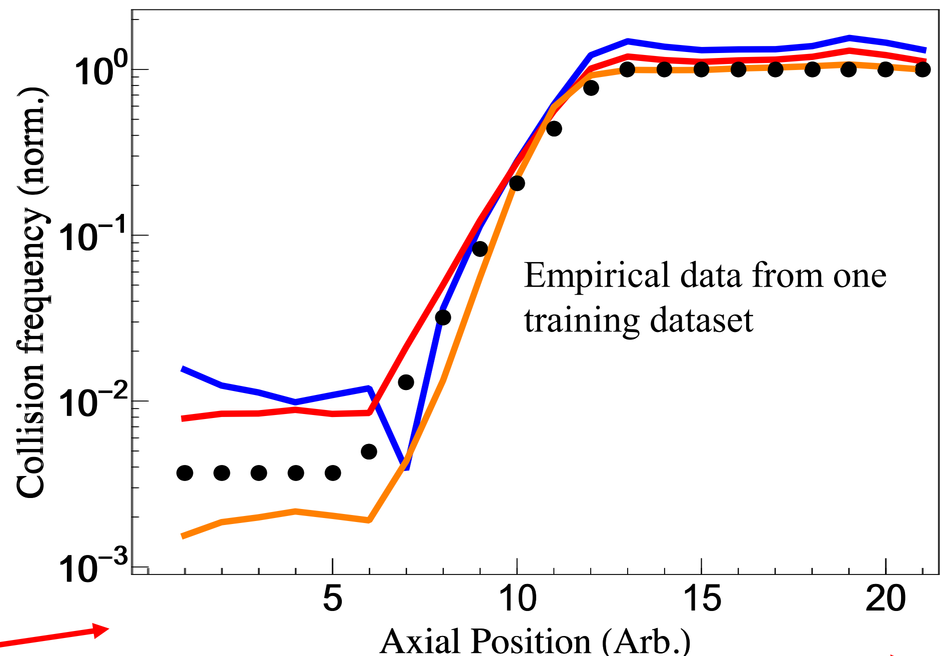

Note: model collision frequency independent of position

B. Jorns., *PSST,* **27** (10), 2018

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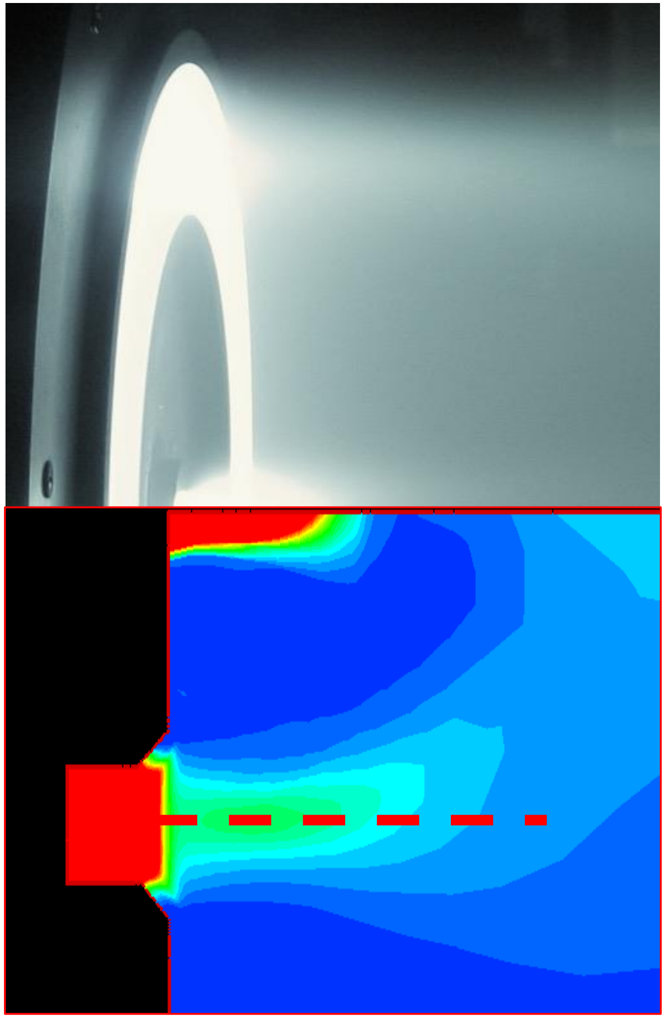

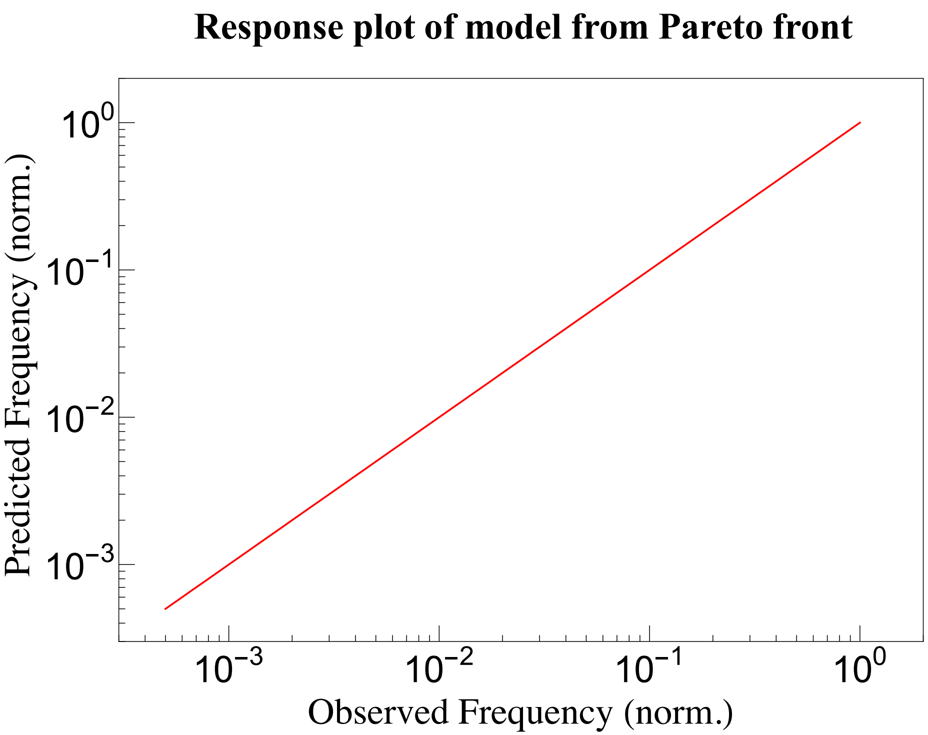

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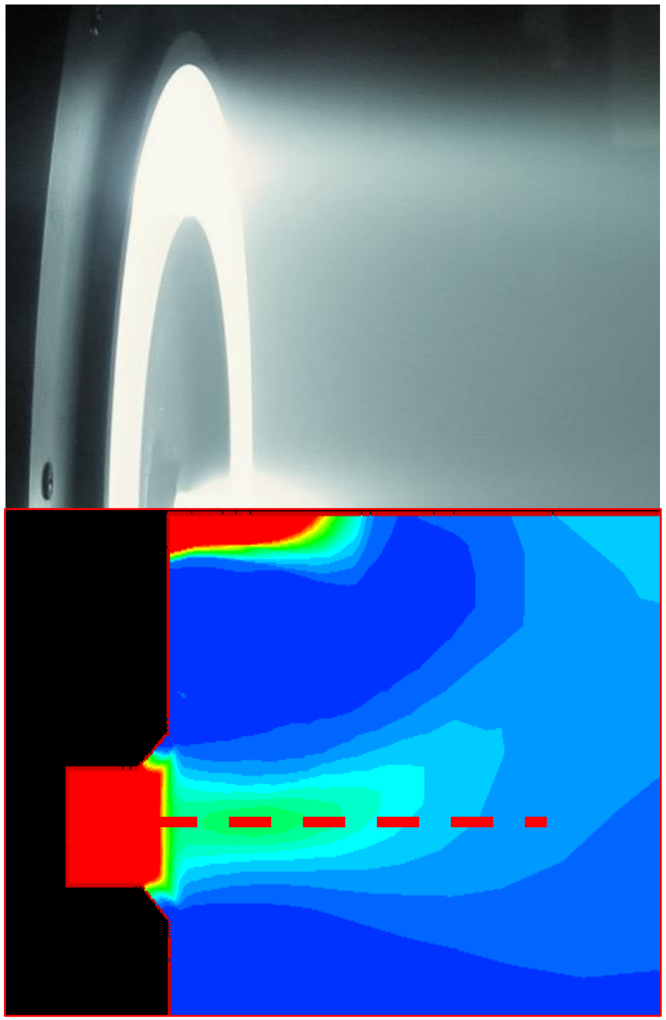
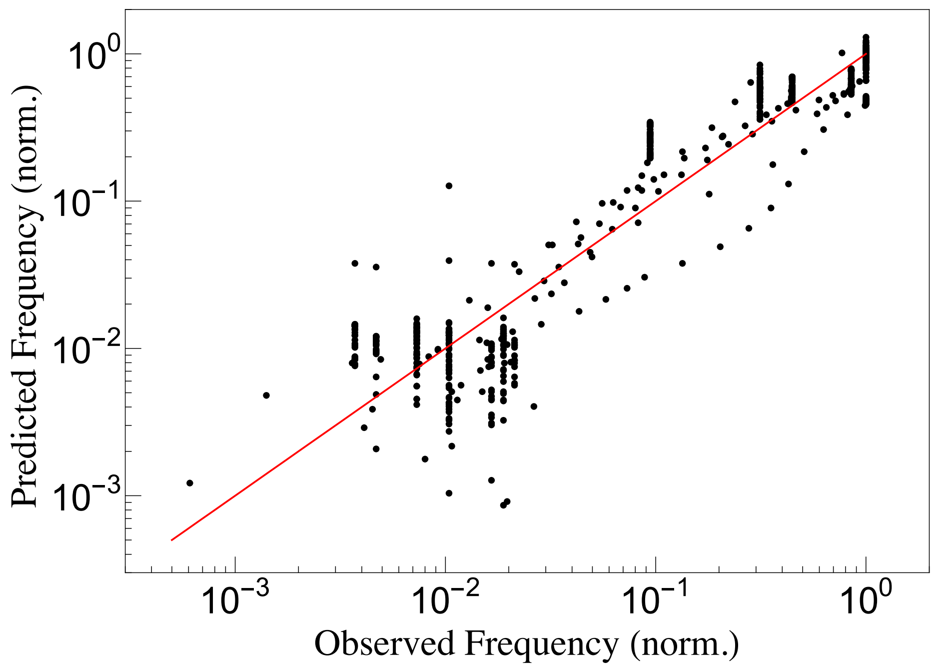


Response plot of model from Pareto front



Correspondence over four orders of magnitude shows promise of ML regression

B. Jorns., *PSST,* **27** (10), 2018

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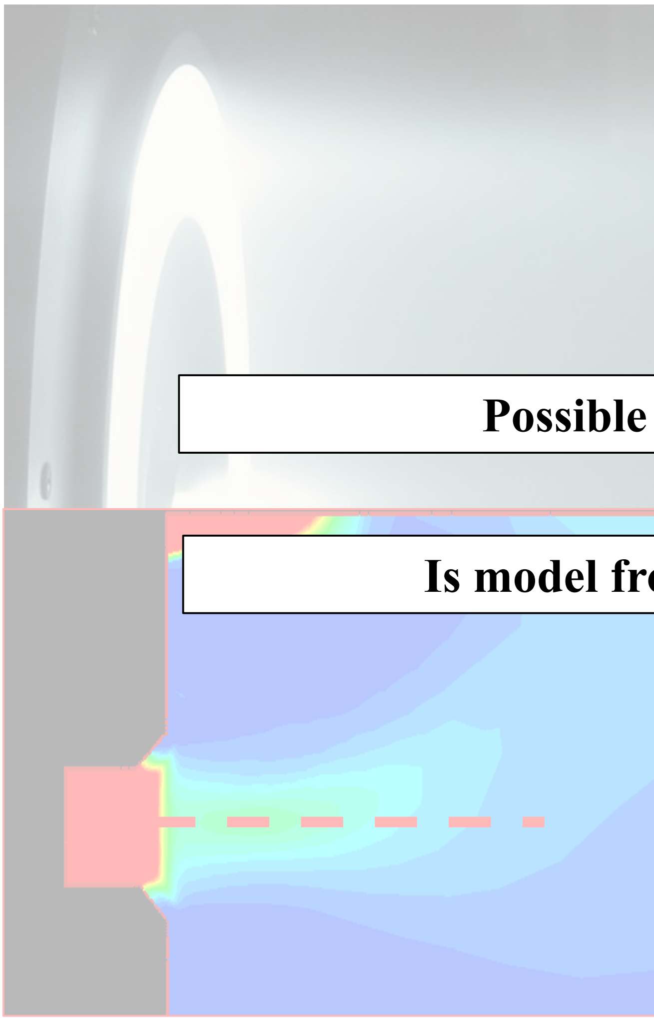
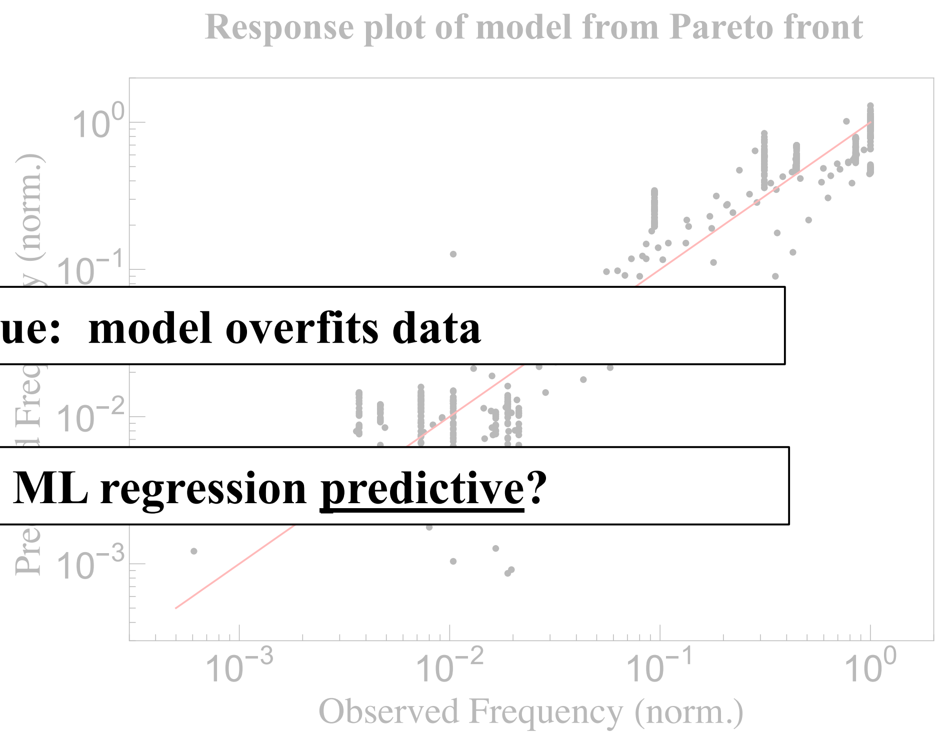


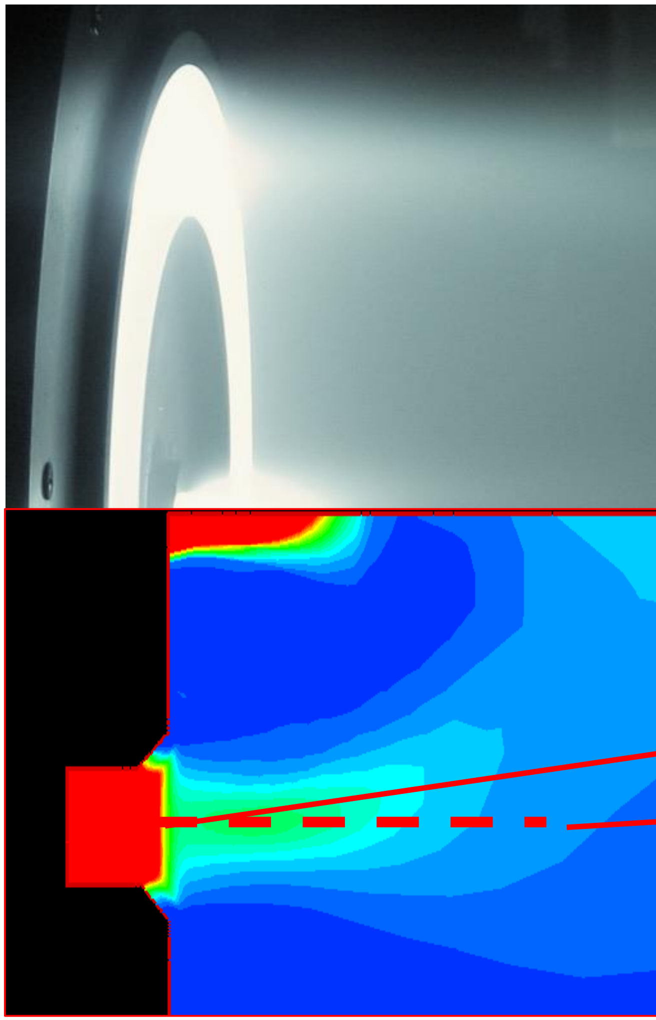


Possible issue: model overfits data

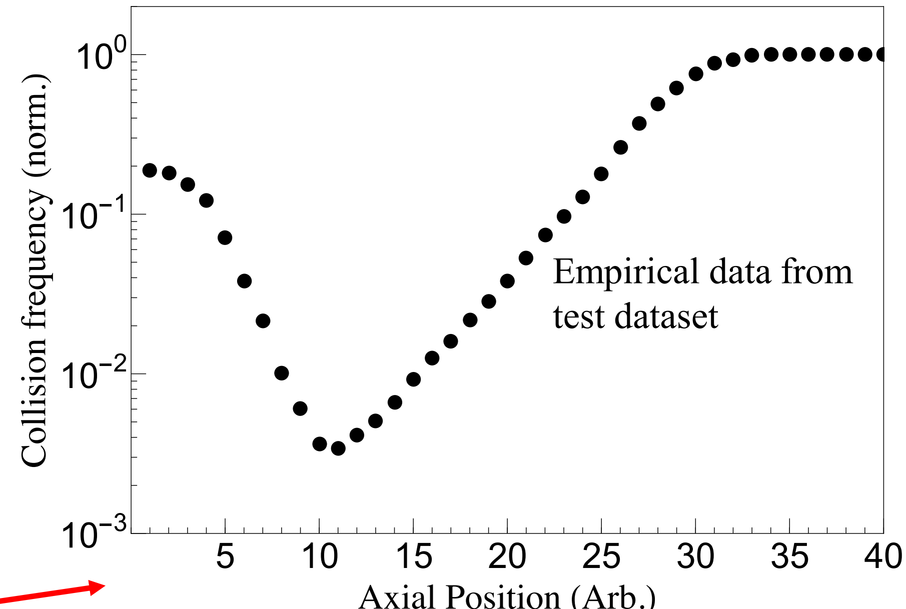
Is model from ML regression <u>predictive</u>?

Correspondence over four orders of magnitude shows promise of ML regression

Predictive capability of model



Data from thruster not included in training dataset



Note: model collision frequency independent of position

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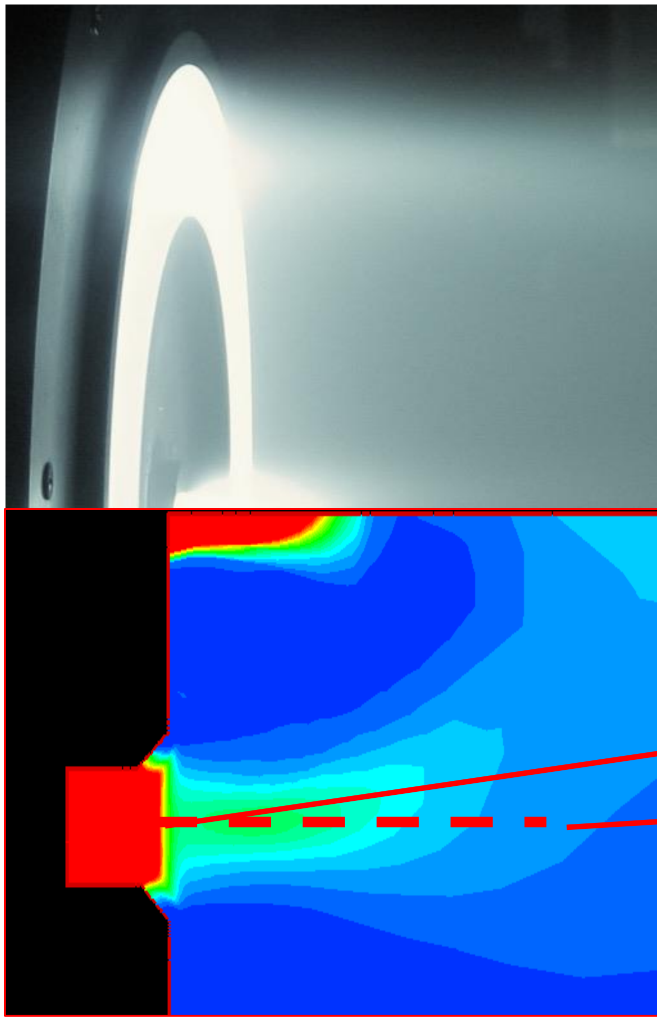

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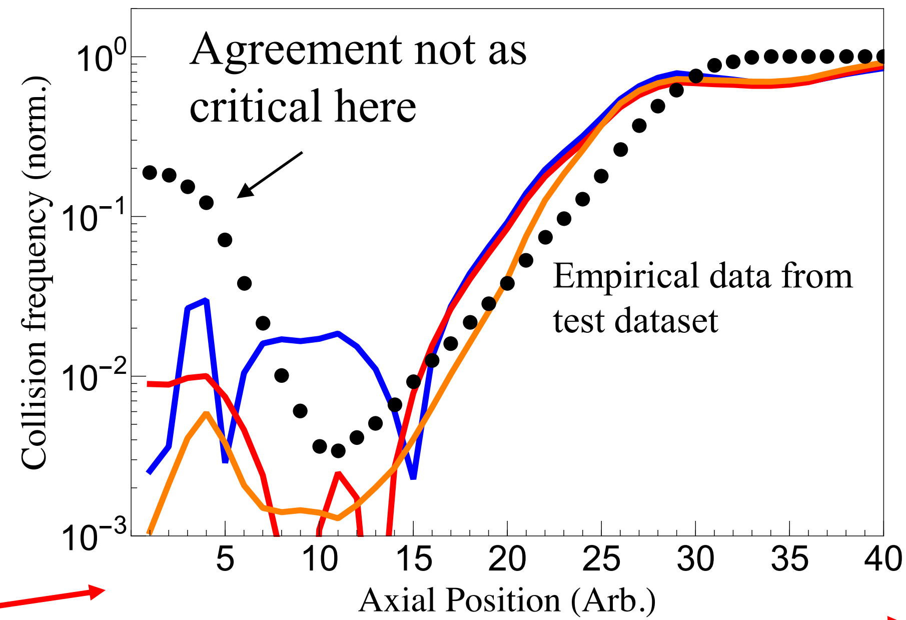

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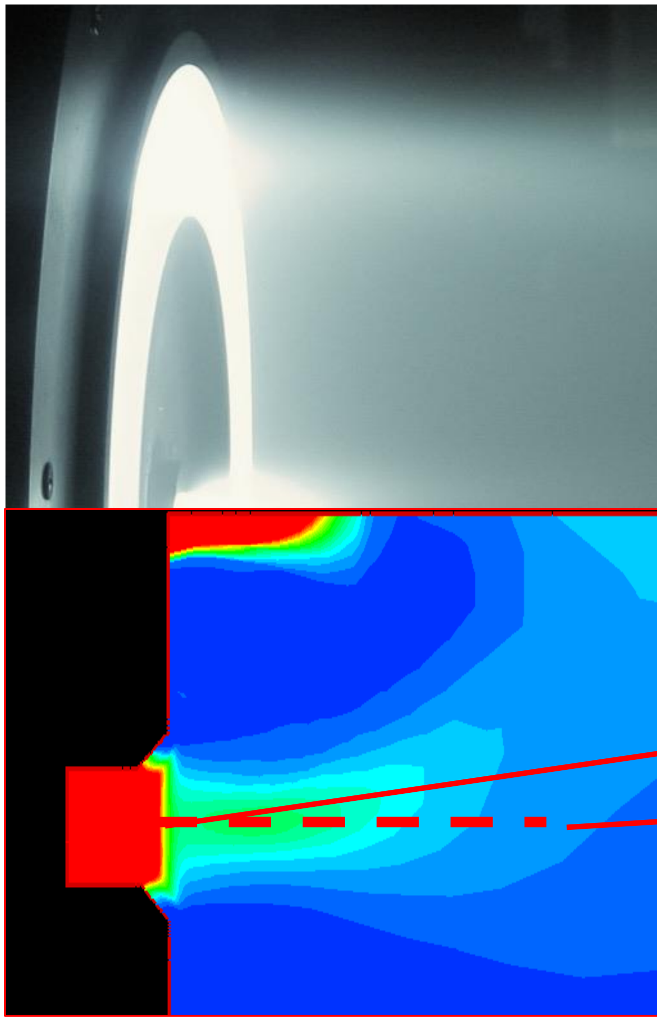
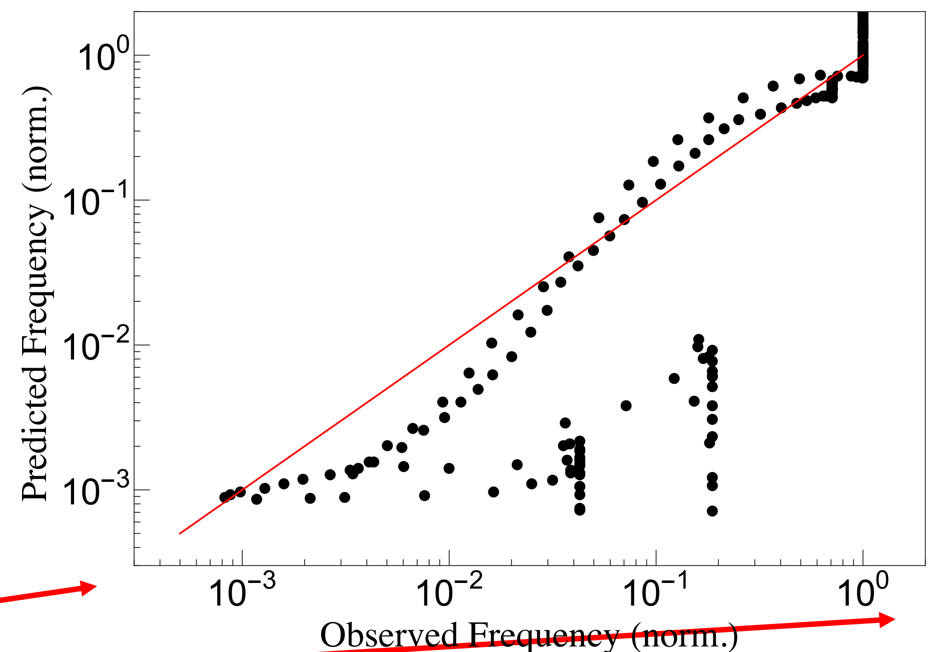
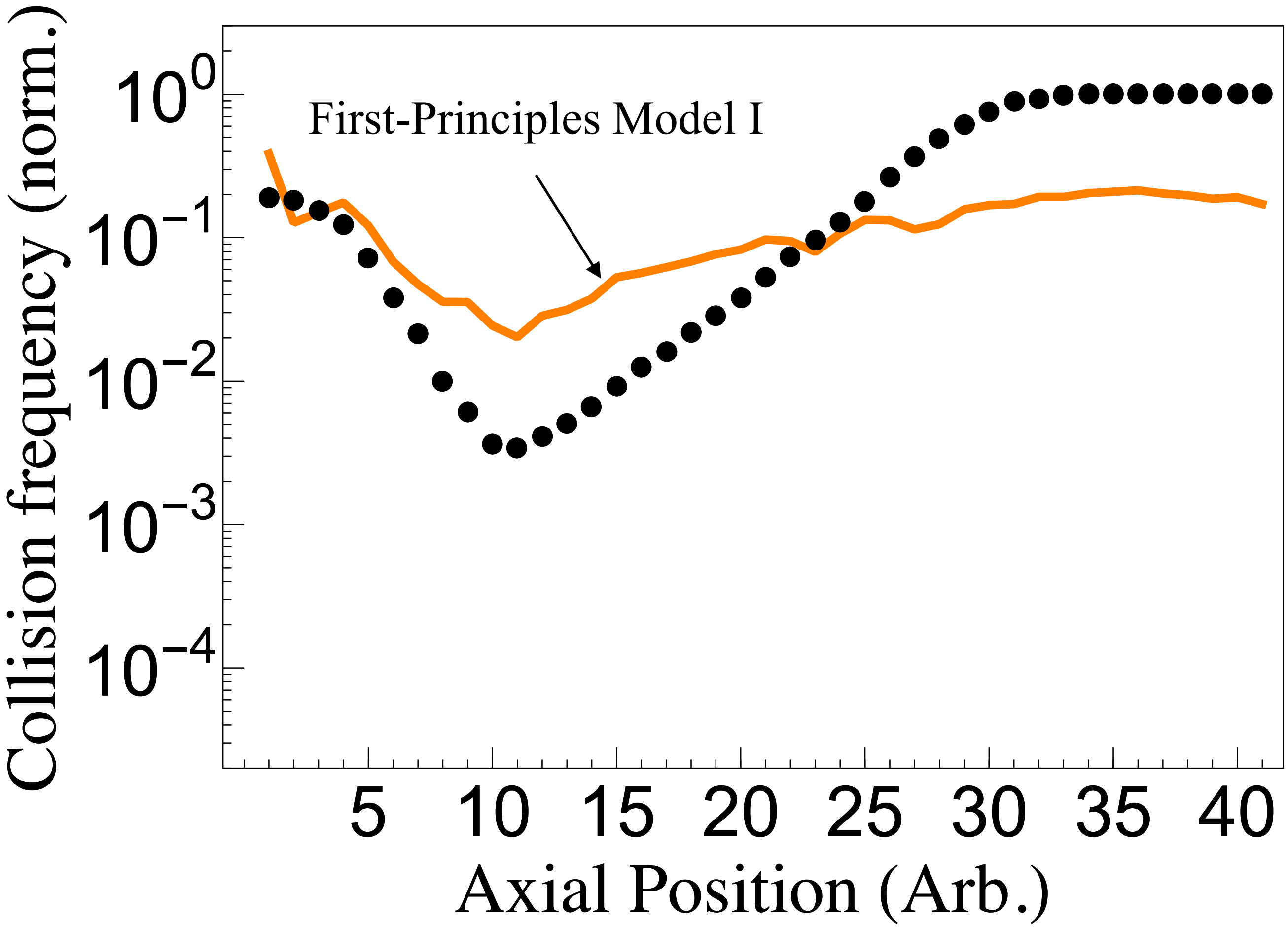


Response plot of ML model to test data



Even though ML model is fit to other data, it can predict collision frequency in new thruster and operating condition

Comparison of ML to first-principles models



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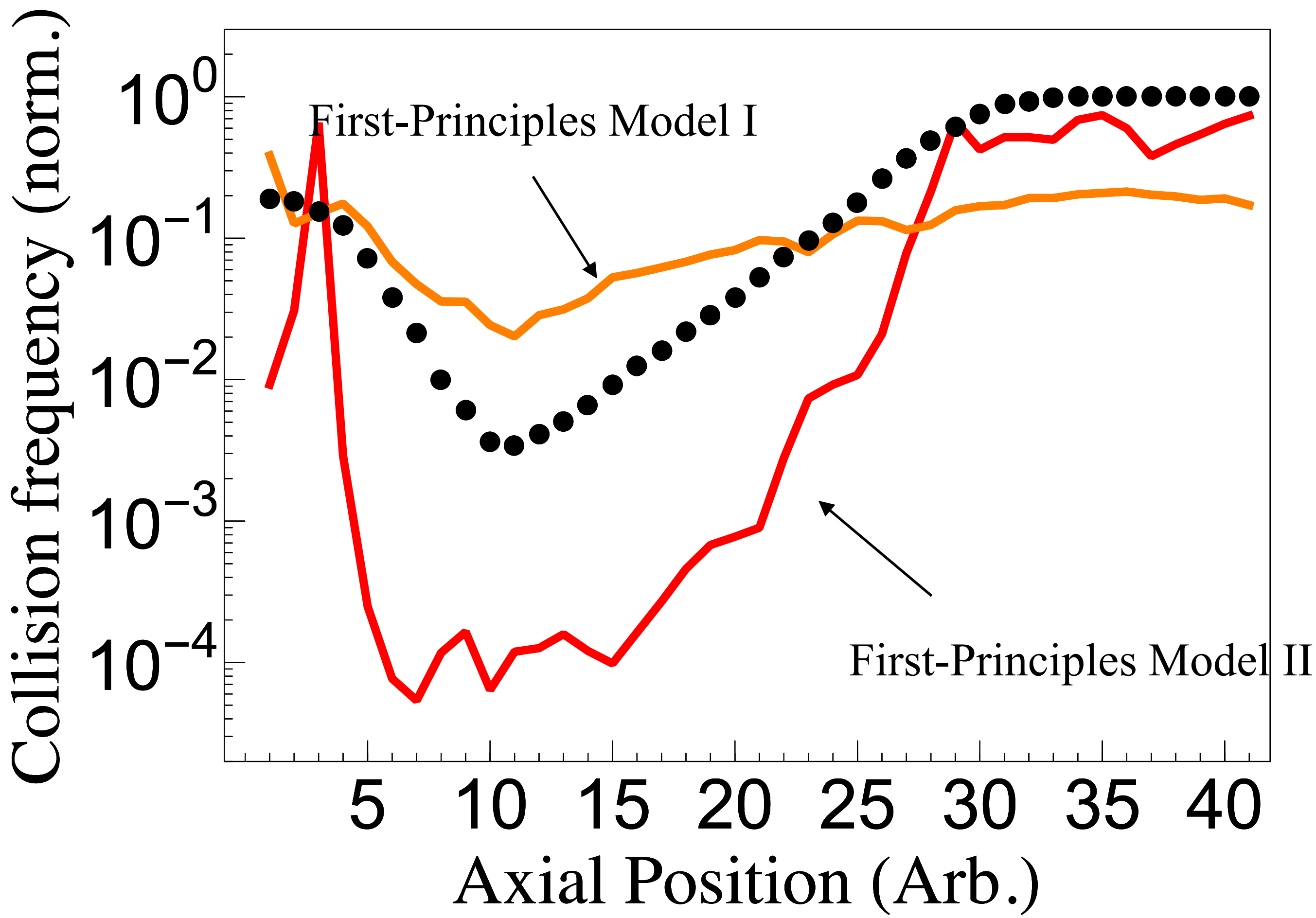

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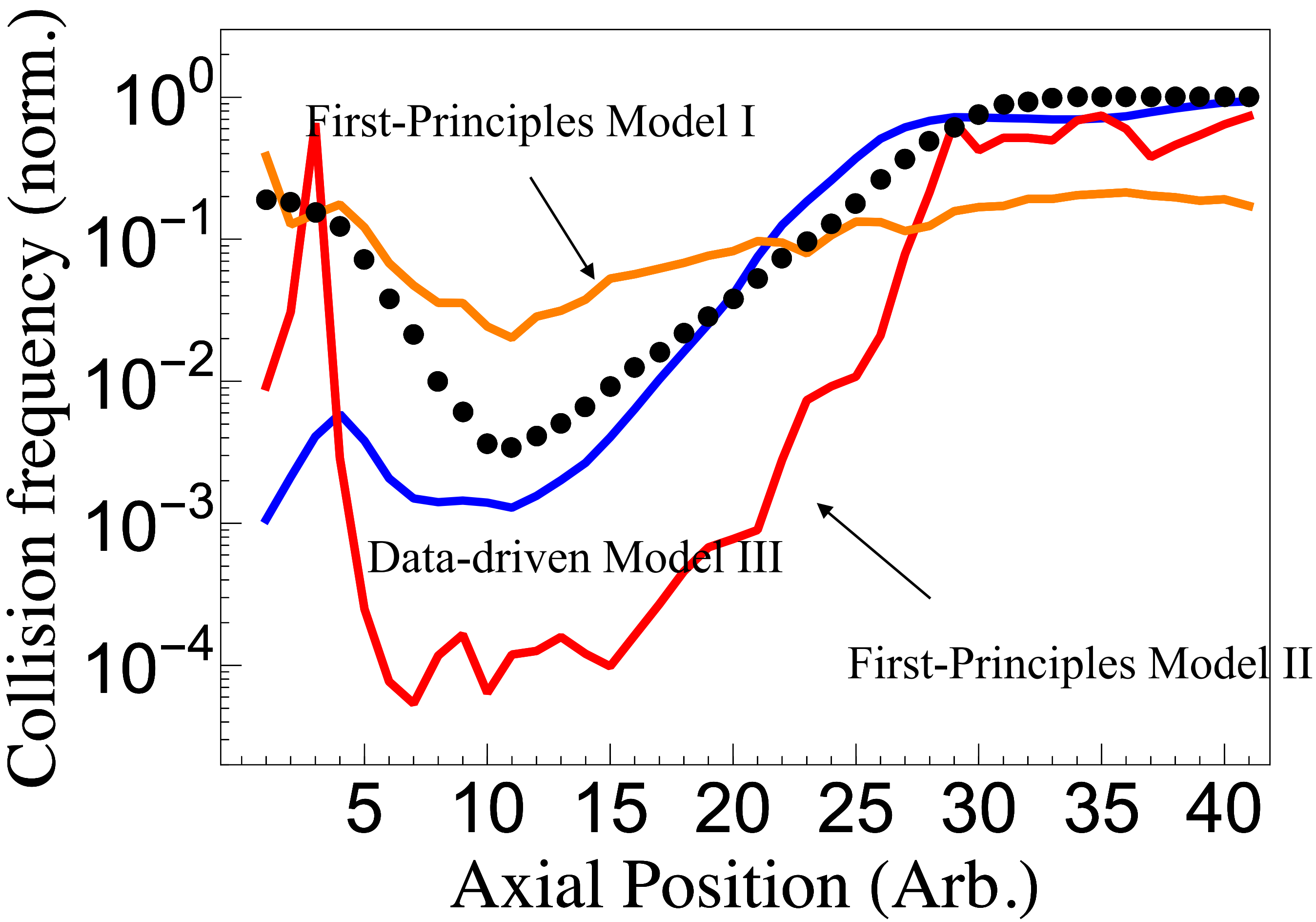
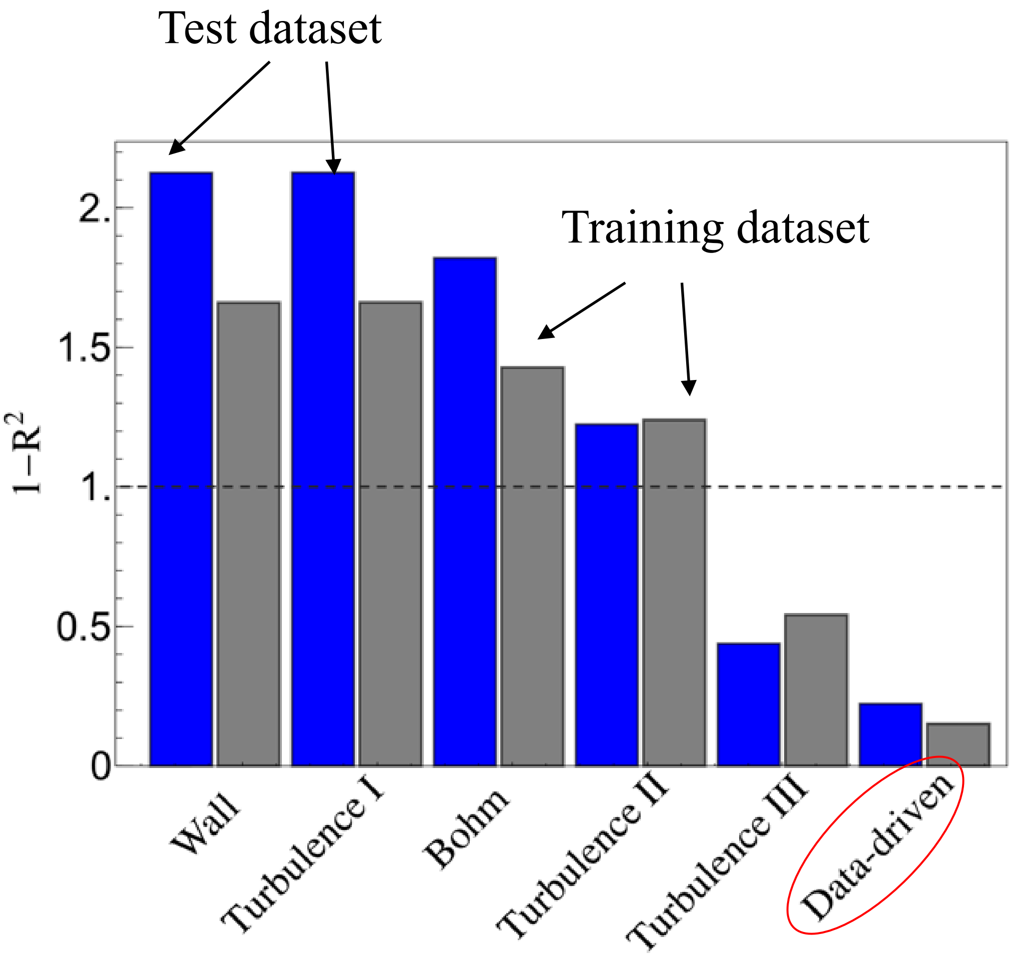


ML model has best correspondence and predictive capability of proposed closures

Comparison of ML to first-principles models



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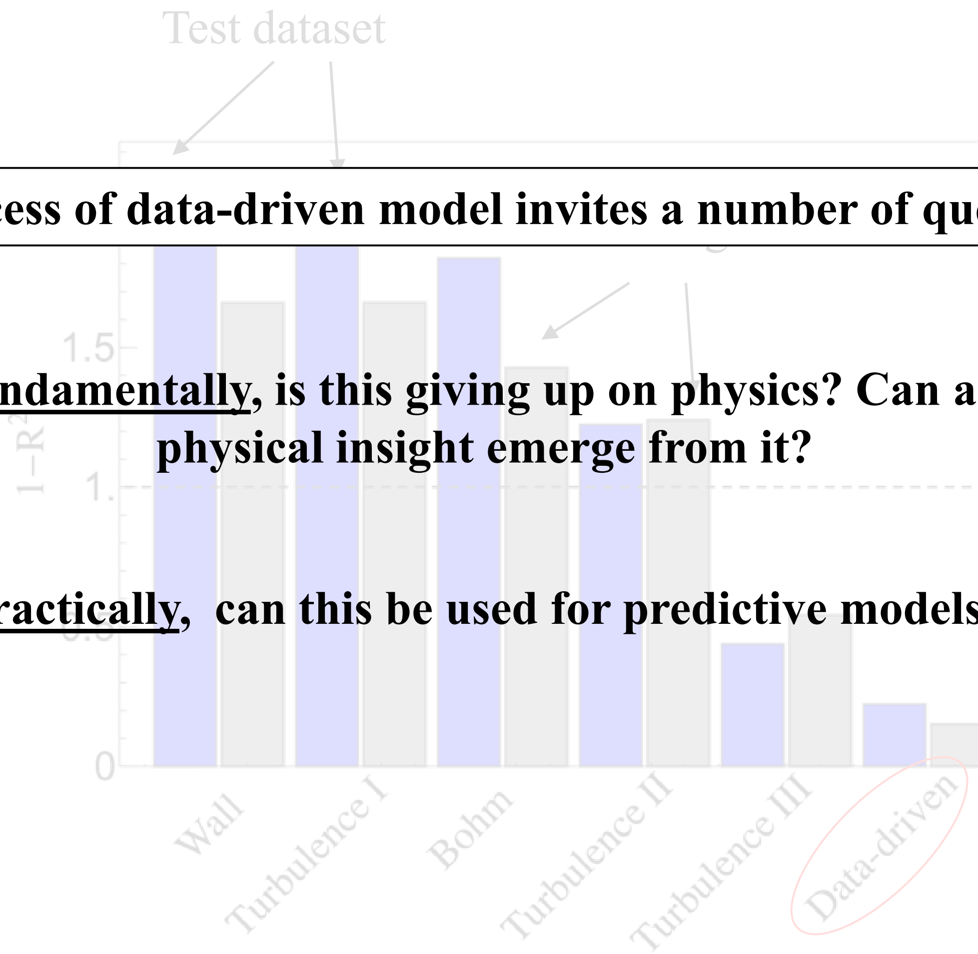

Success of data-driven model invites a number of questions

<u>Fundamentally</u>, is this giving up on physics? Can any physical insight emerge from it?

<u>Practically</u>, can this be used for predictive models?

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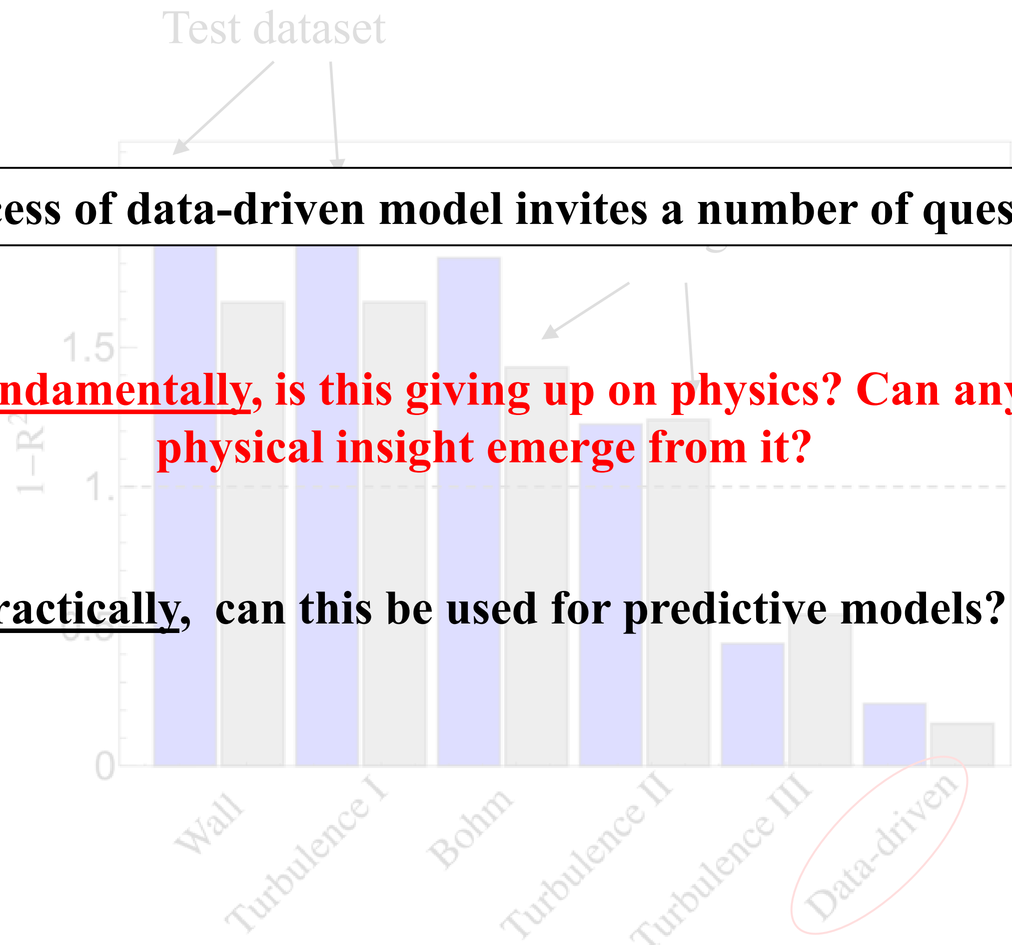

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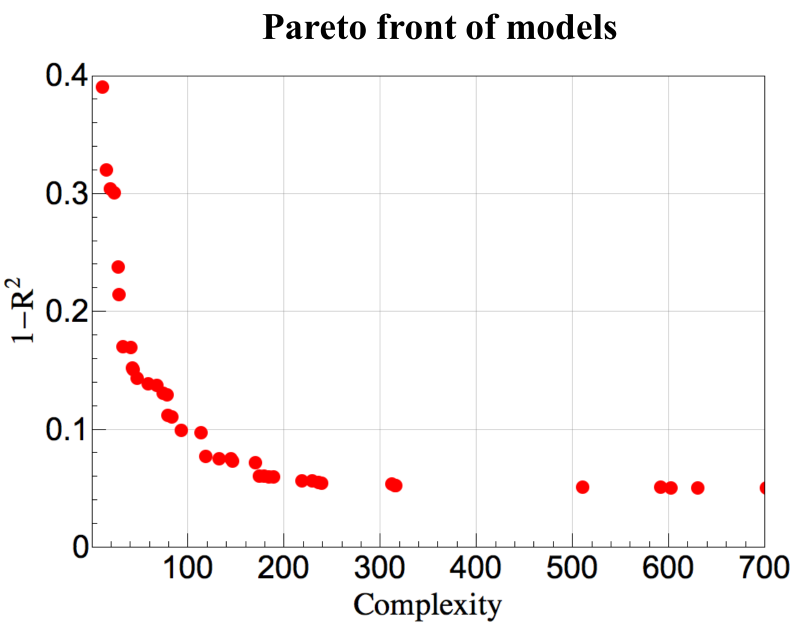
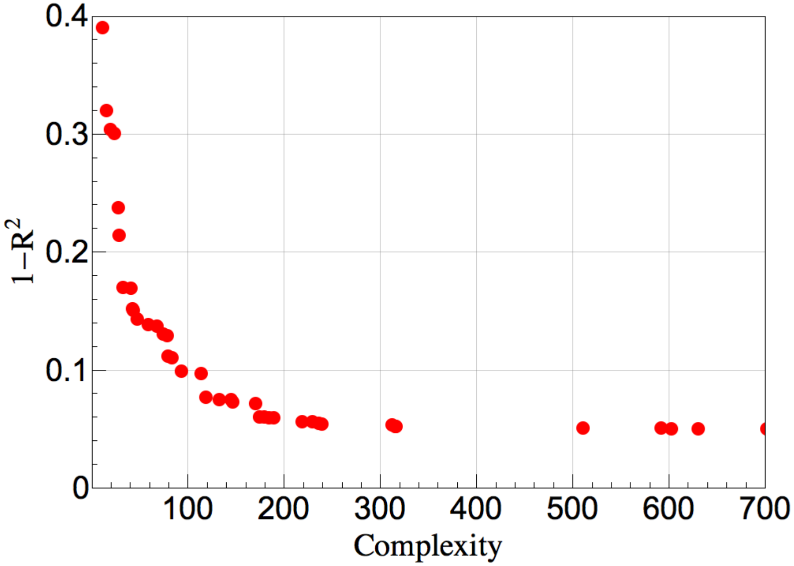
Physical insight



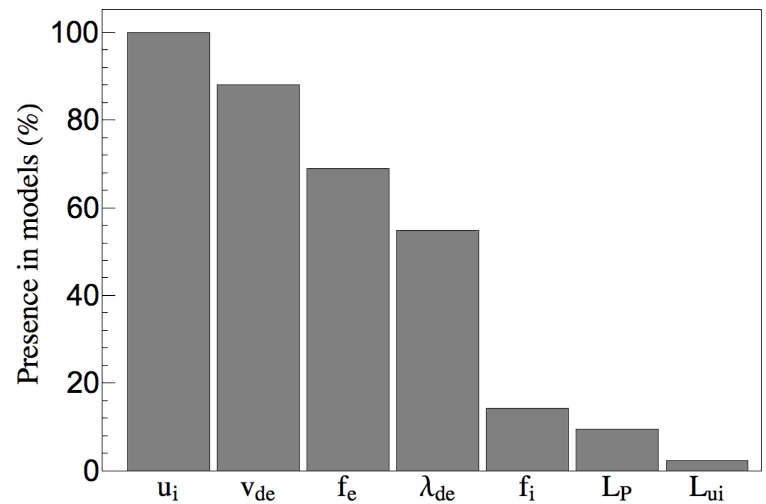
From these models, are there are any variables that are more common than others?

Physical insight

Pareto front of models



Frequency of variable appearance in best models



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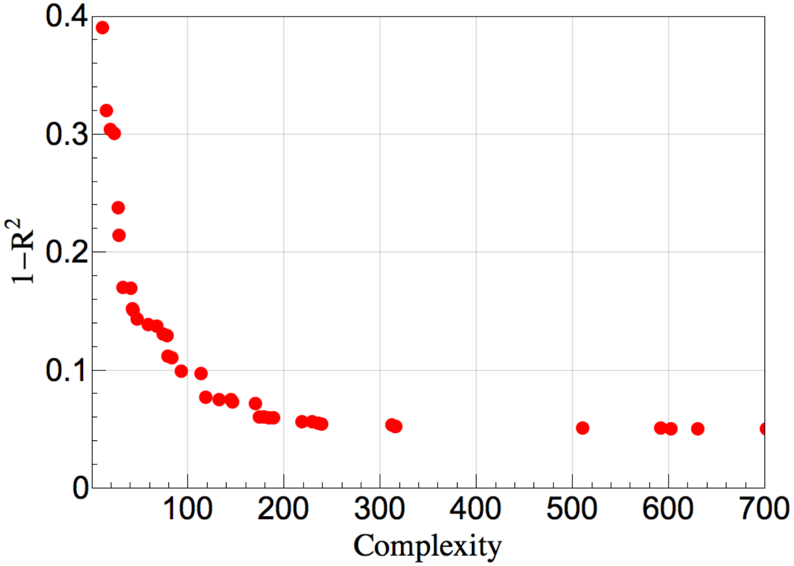

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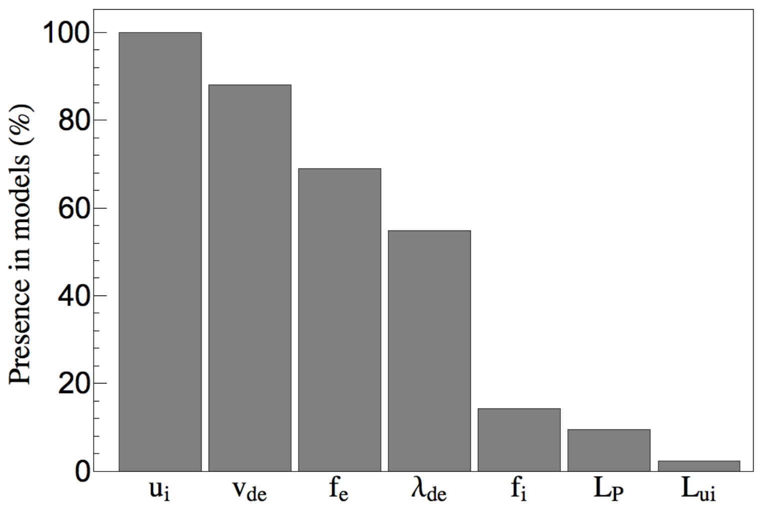
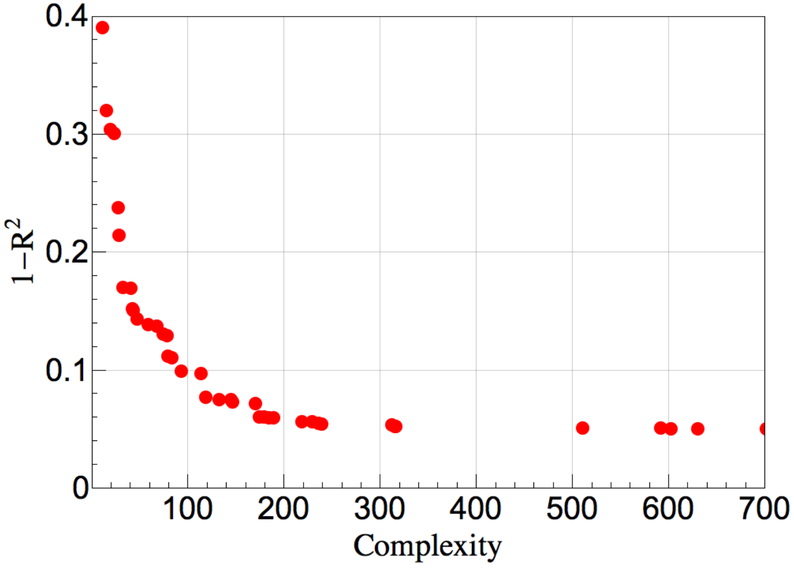


Ion drift and Hall drift dominant variables

Physical insight

Pareto front of models



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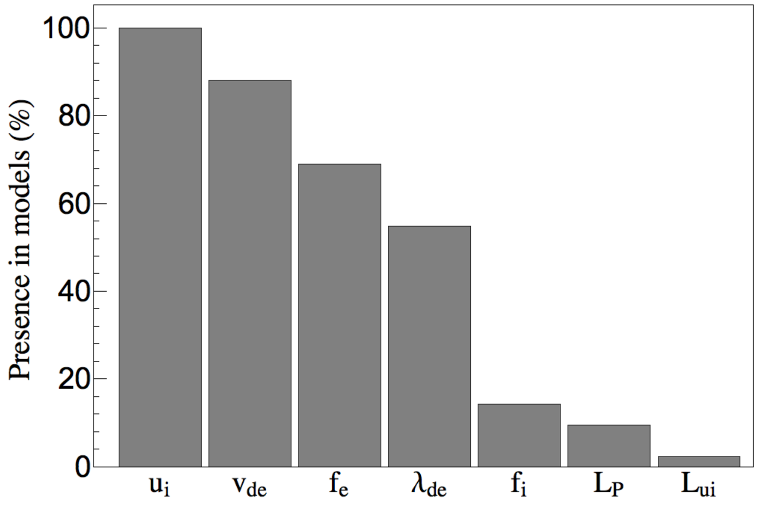
Frequency of variable appearance in best models



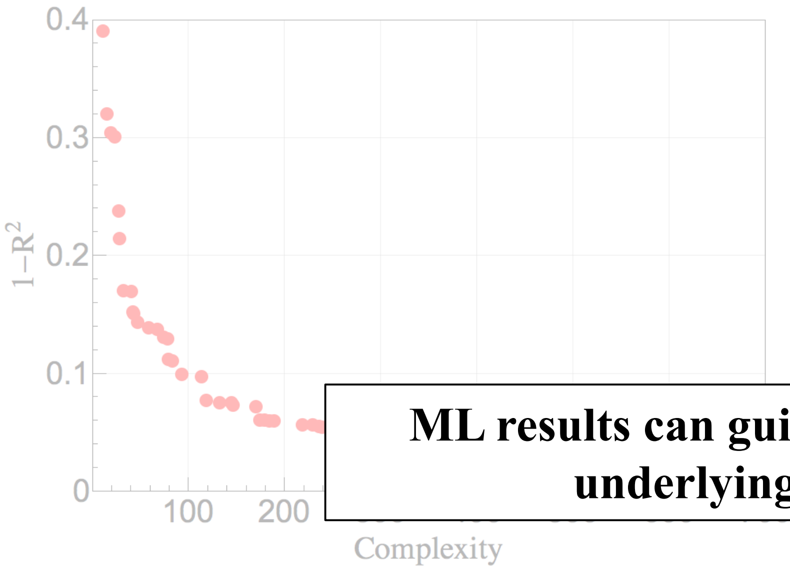
Ion drift and Hall drift dominant variables

Search for a first-principles mechanism that depends on these parameters

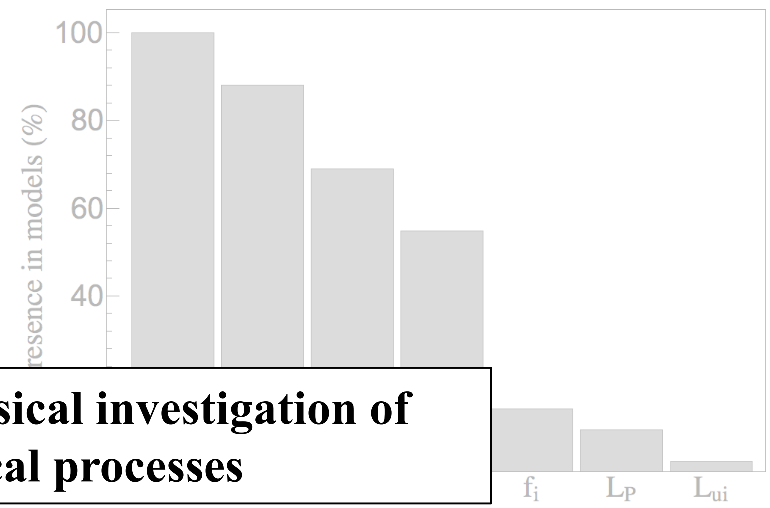
Electron cyclotron drift instability one example

Physical insight

Pareto front of models



Frequency of variable appearance in best models



ML results can guide physical investigation of underlying physical processes

From these models, are there are any variables that are more common than others?

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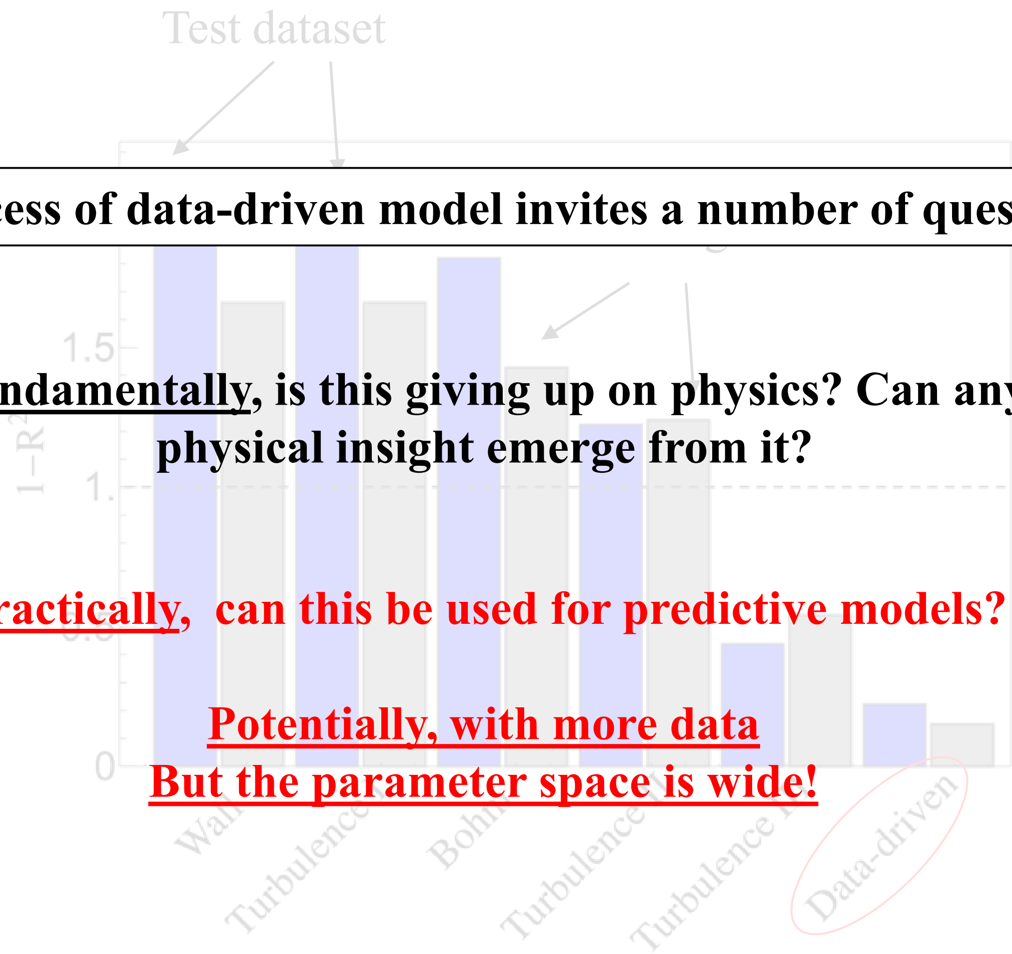
Comparison of ML to first-principles models

Success of data-driven model invites a number of questions



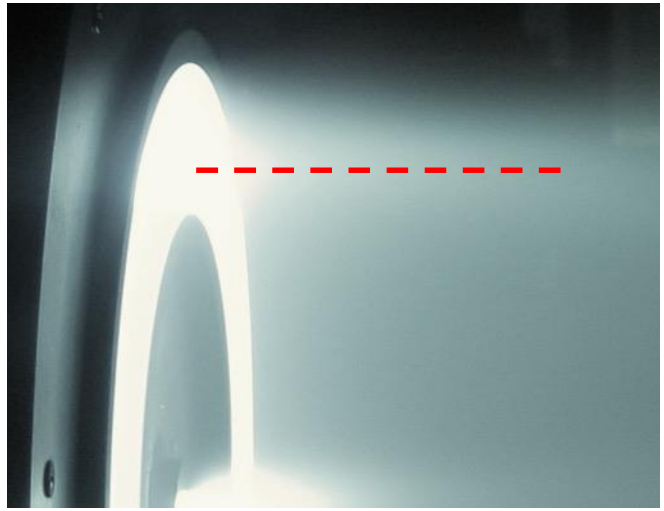
<u>Fundamentally</u>, is this giving up on physics? Can any physical insight emerge from it?

<u>Practically</u>, can this be used for predictive models?

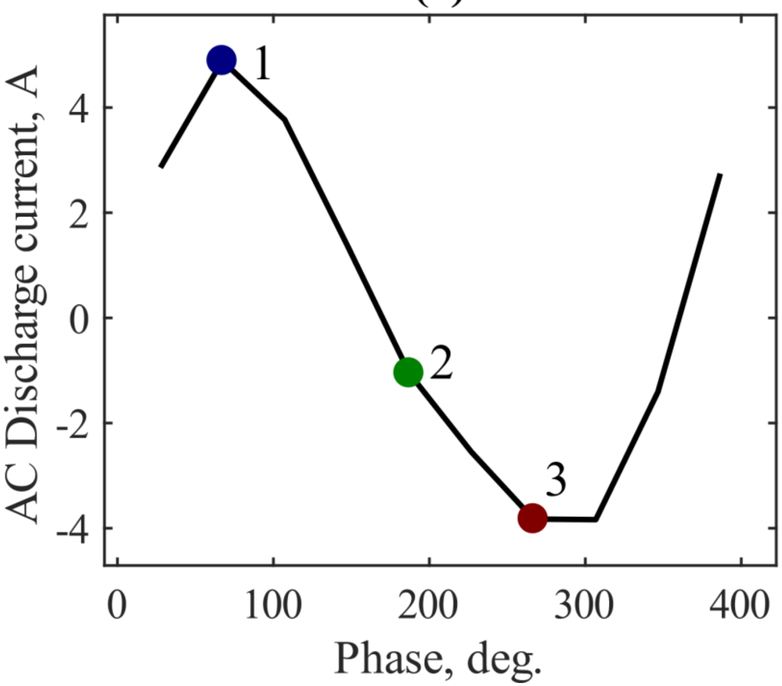
<u>Potentially, with more data</u>
<u>But the parameter space is wide!</u>

ML model has best correspondence and predictive capability of proposed closures

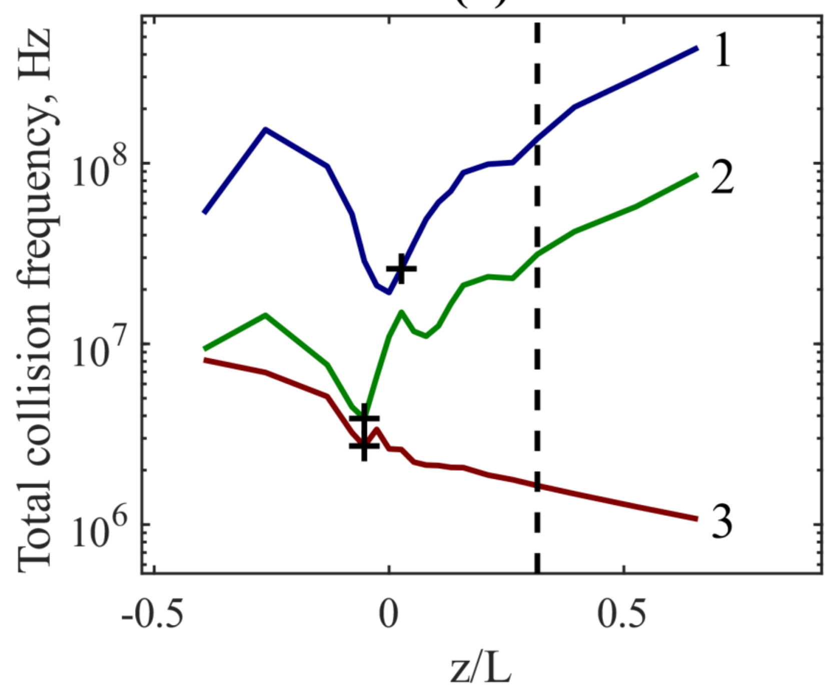
Generating additional data on transport in Hall thrusters



(a)



(b)



Summary

- Fluid models are attractive tool for modeling Hall effect thrusters
- Need to account for known anomalous electron transport in these models with a type of closure: typically anomalous effects represented with scalar collision frequency (or mobility)
- Data-driven, ML methods can be employed to find functional form for this anomalous collision frequency
- Predictions from ML results yield
  - Improved results compared to first-principles models for anomalous collision frequency
  - ML algorithm also yields physical insight into dominant terms governing transport
- **ML is a promising path forward for closing anomalous electron transport problem. Predictive capability has applications ranging from predictive design to qualification through analysis.**
- **On-going challenges include**
  - Extrapolation
  - Data-generation